

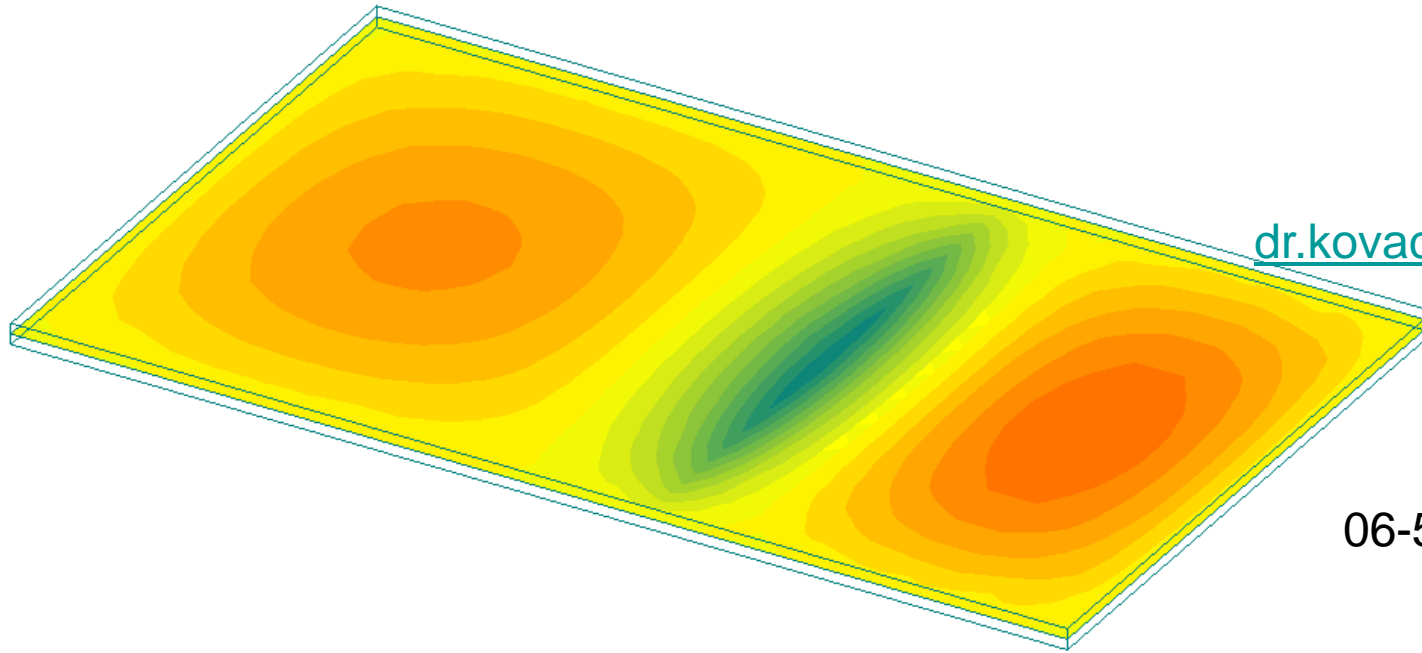
Reinforced Concrete Structures II.

II.

Vasbetonszerkezetek II.

- Kétirányban teherviselő lemez nyomatókai - Tartósáv módszer -

Dr. Kovács Imre PhD
tanszékvezető
főiskolai tanár



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WEB:
<http://epitotsz.mk.unideb.hu/>

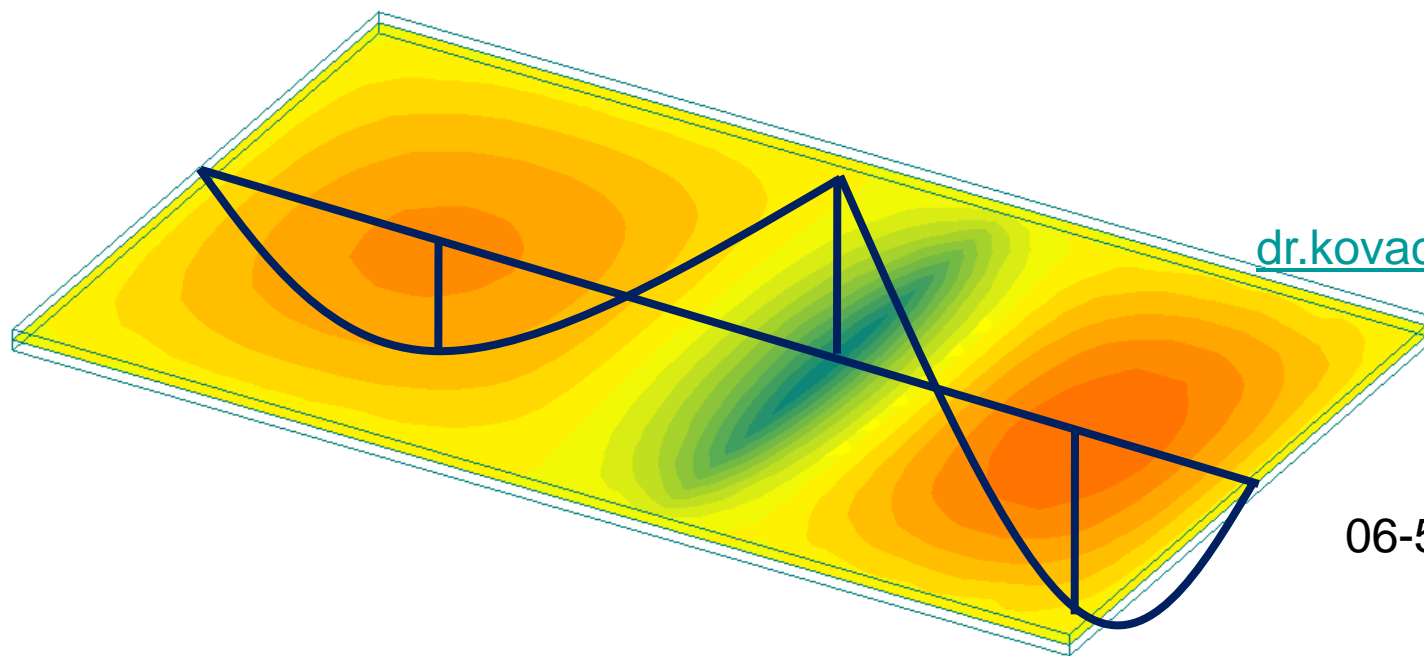
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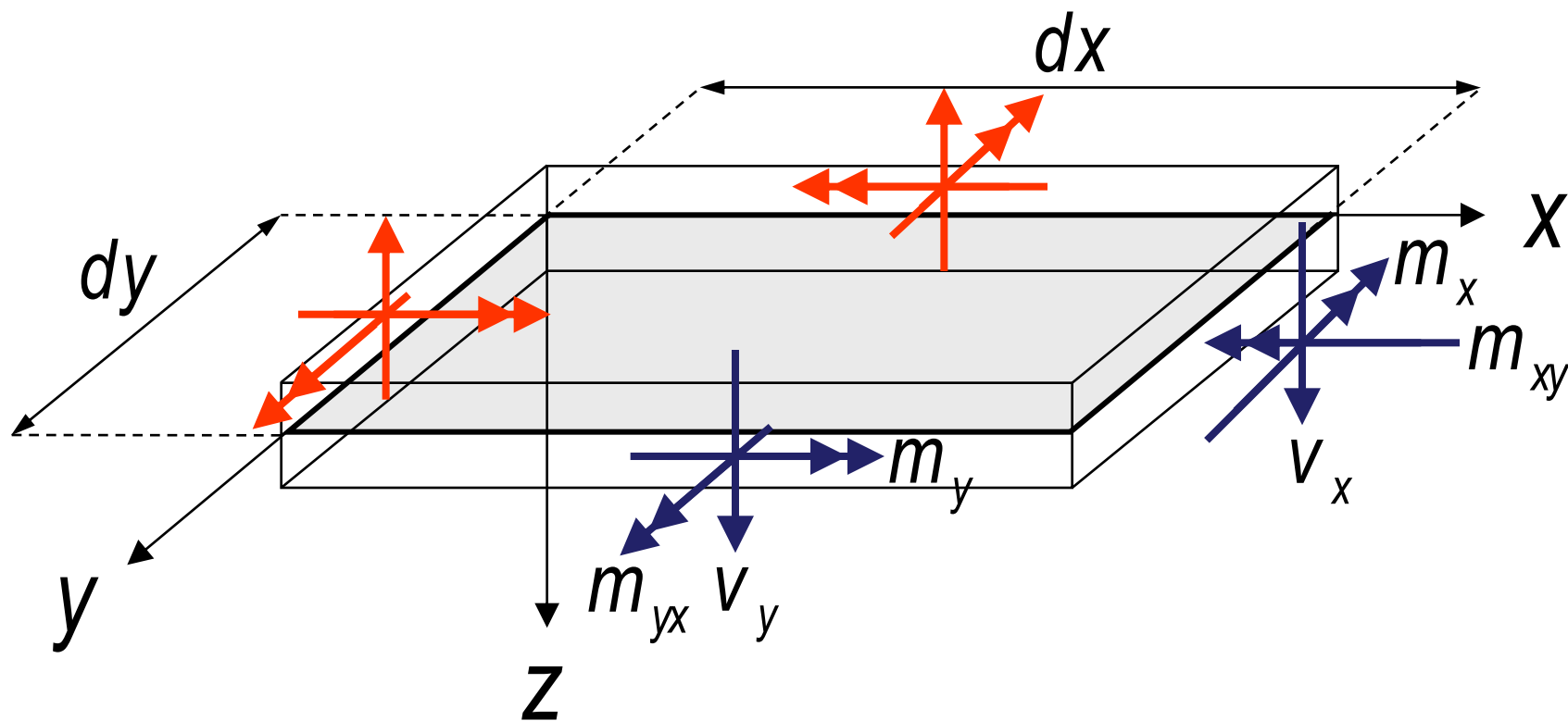
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Bending, shear and torsion of a Kirchhoff-type slab

Kirchhoff-féle lemez igénybevételei



Bending, shear and torsion of a Kirchoff-type slab

Kirchhoff-féle lemez igénybevételei

x és y irányú fajlagos hajlítónyomatékok

$$m_x = -K \cdot \left(\frac{\partial^2 w}{\partial x^2} + \mu_c \cdot \frac{\partial^2 w}{\partial y^2} \right)$$

$$m_y = -K \cdot \left(\frac{\partial^2 w}{\partial y^2} + \mu_c \cdot \frac{\partial^2 w}{\partial x^2} \right)$$

fajlagos csavarónyomatékok

$$m_{xy} = m_{yx} = K \cdot (1 - \mu_c) \cdot \frac{\partial^2 w}{\partial x \partial y}$$

x és y irányú fajlagos nyíróerők

$$v_x = -K \cdot \frac{\partial}{\partial x} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)$$

$$v_y = -K \cdot \frac{\partial}{\partial y} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)$$

a lemez hajlítómerevsége

$$K = \frac{E \cdot v^3}{12 \cdot (1 - \mu_c^2)}$$

Rugalmissági
modulus

Harántnyúlási tényező
(Poisson szám reciproka)
0,15~0,20

Equilibrium of a Kirchoff-type slab

Kirchhoff-féle lemez egyensúlya

Lemezelem egyensúlyi egyenlete:

$$\frac{\partial^2 m_x}{\partial x^2} + 2 \cdot \frac{\partial^2 m_{xy}}{\partial x \partial y} + \frac{\partial^2 m_y}{\partial y^2} = -q(x, y)$$

Fizikai (anyag) egyenletek:

$$\sigma_x = \frac{E}{1 - \mu_c^2} \cdot (\varepsilon_x + \mu_c \cdot \varepsilon_y)$$

$$\tau_{xy} = \frac{E}{2 \cdot (1 + \mu_c)} \cdot \gamma_{xy}$$

$$\sigma_y = \frac{E}{1 - \mu_c^2} \cdot (\varepsilon_y + \mu_c \cdot \varepsilon_x)$$

Kompatibilitási (összeférhetőségi) egyenletek

$$\varepsilon_x = -z \cdot \frac{\partial^2 w}{\partial x^2}$$

$$\gamma_{xy} = -2 \cdot z \cdot \frac{\partial^2 w}{\partial x \partial y}$$

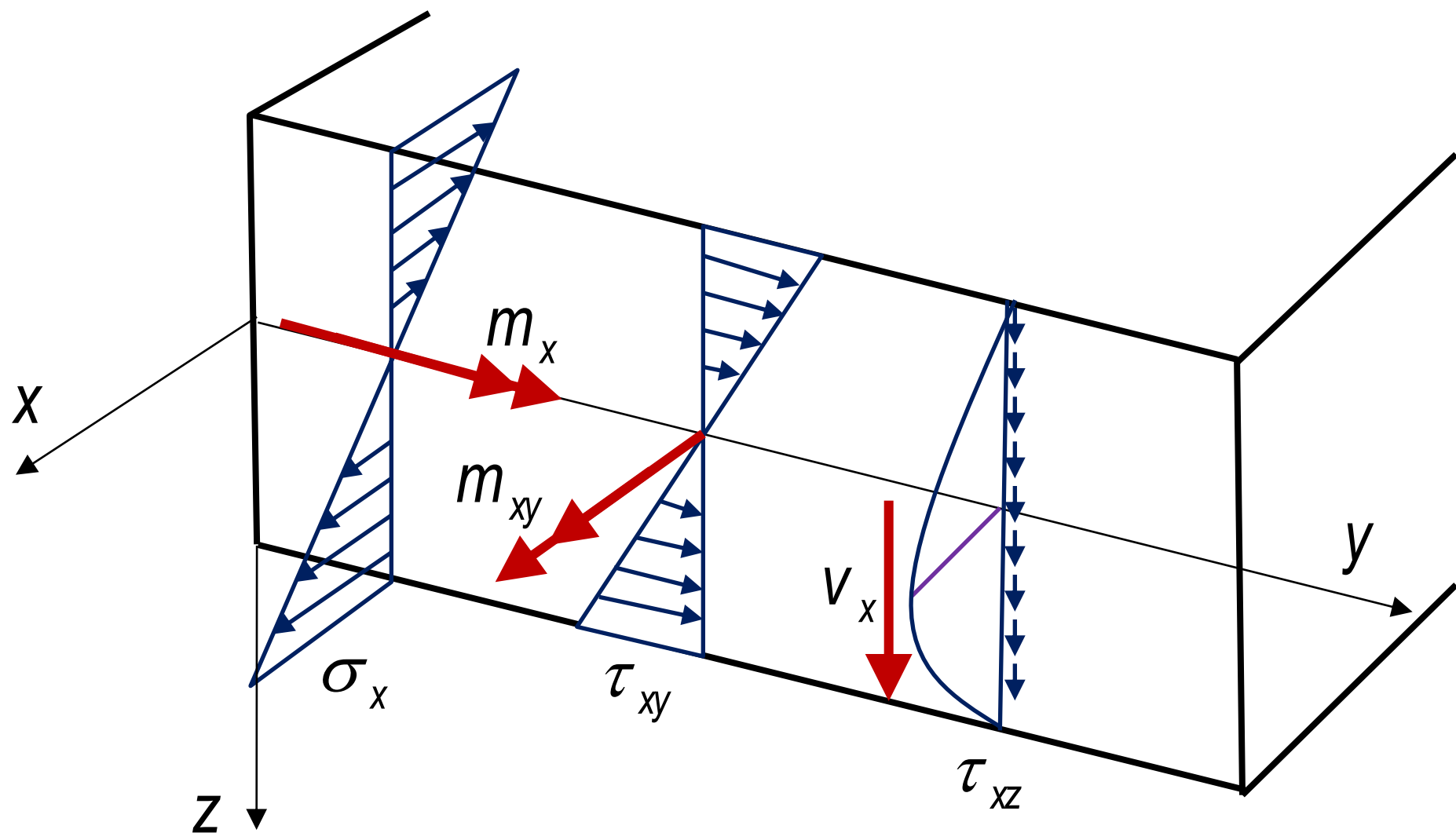
$$\varepsilon_y = -z \cdot \frac{\partial^2 w}{\partial y^2}$$

Lemez egyenlet:

$$\frac{\partial^4 w}{\partial x^4} + 2 \cdot \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q(x, y)}{k}$$

Stresses of a Kirchhoff-type slab

Kirchhoff-féle lemez feszültségei



Equation of the elastic slab

Rugalmas lemez egyenlete

Lemez egyenlet:

$$\frac{\partial^4 w}{\partial x^4} + 2 \cdot \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q(x, y)}{k}$$

Lagrange féle formula (1812)

$$\Delta \Delta w = \frac{q(x, y)}{k}$$

Kétváltozós Laplace operátor

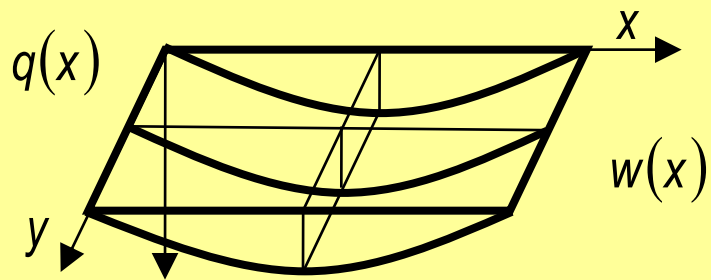
$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Kétváltozós, negyedrendű (biharmonikus) differenciálegyenlet matematikai határozottságához minden perempontban két peremfeltételt kell előírni!

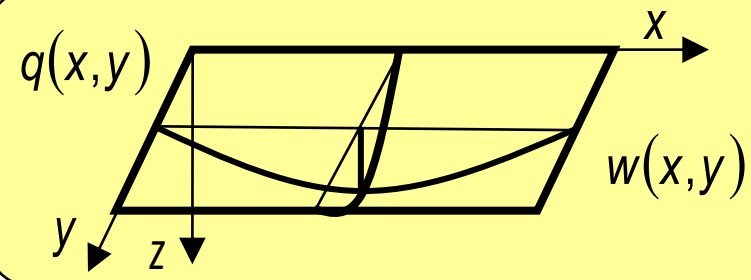
Cylindrical bending of slab

Lemezek hengeres hajlítása

Lemezek hengeres hajlítása



Kirchhoff-féle lemez (általános)



A teher és a hajlítónyomaték közötti differenciális összefüggés

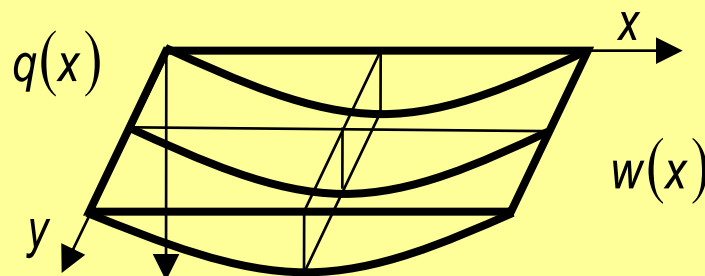
$$\frac{\partial^2 m_x}{\partial x^2} = -q(x)$$

$$\frac{\partial^2 m_x}{\partial x^2} + 2 \cdot \frac{\partial^2 m_{xy}}{\partial x \partial y} + \frac{\partial^2 m_y}{\partial y^2} = -q(x,y)$$

One-way slab

Egyirányban teherviselő lemez

Egy irányban teherviselő lemez



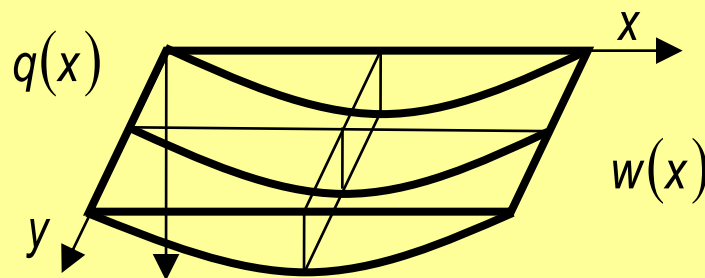
$$m_x = -K \cdot \left(\frac{\partial^2 w}{\partial x^2} + \mu_c \cdot \frac{\partial^2 w}{\partial y^2} \right)$$

$$m_y = -K \cdot \left(\frac{\partial^2 w}{\partial y^2} + \mu_c \cdot \frac{\partial^2 w}{\partial x^2} \right)$$

One-way slab

Egyirányban teherviselő lemez

Egy irányban teherviselő lemez



$$m_x = -K \cdot \left(\frac{\partial^2 w}{\partial x^2} + \mu_c \cdot \frac{\partial^2 w}{\partial y^2} \right)$$

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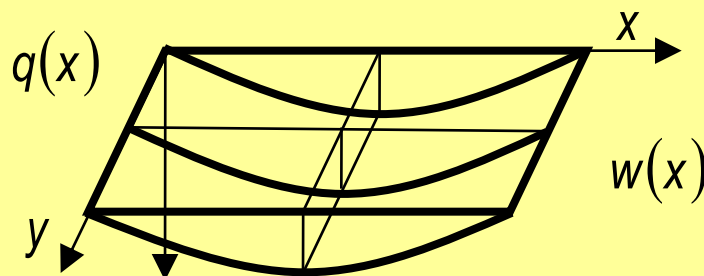
$$m_x = -K \cdot \left(\frac{\partial^2 w}{\partial x^2} + \mu_c \cdot 0 \right)$$

$$m_y = -K \cdot \left(0 + \mu_c \cdot \frac{\partial^2 w}{\partial x^2} \right)$$

One-way slab

Egyirányban teherviselő lemez

Egy irányban teherviselő lemez



$$m_x = -K \cdot \left(\frac{\partial^2 w}{\partial x^2} + \mu_c \cdot \frac{\partial^2 w}{\partial y^2} \right)$$

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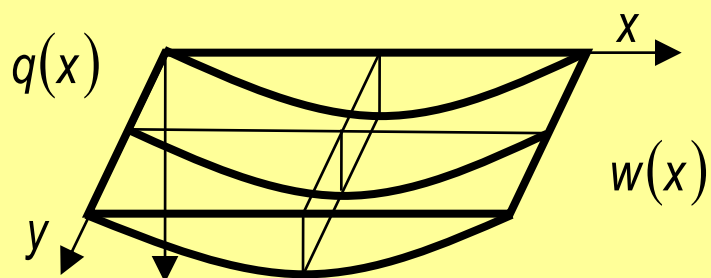
$$m_x = -K \cdot \frac{\partial^2 w}{\partial x^2}$$

$$m_y = -\mu_c \cdot K \cdot \frac{\partial^2 w}{\partial x^2} = \mu_c \cdot m_x$$

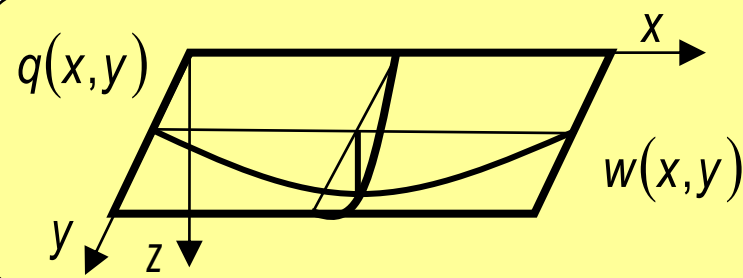
One-way and two-way slab

Egy- és kétirányban teherrelő lemez

Egy irányban teherrelő lemez



Két irányban teherrelő lemez



Lemezelmélet alapján:

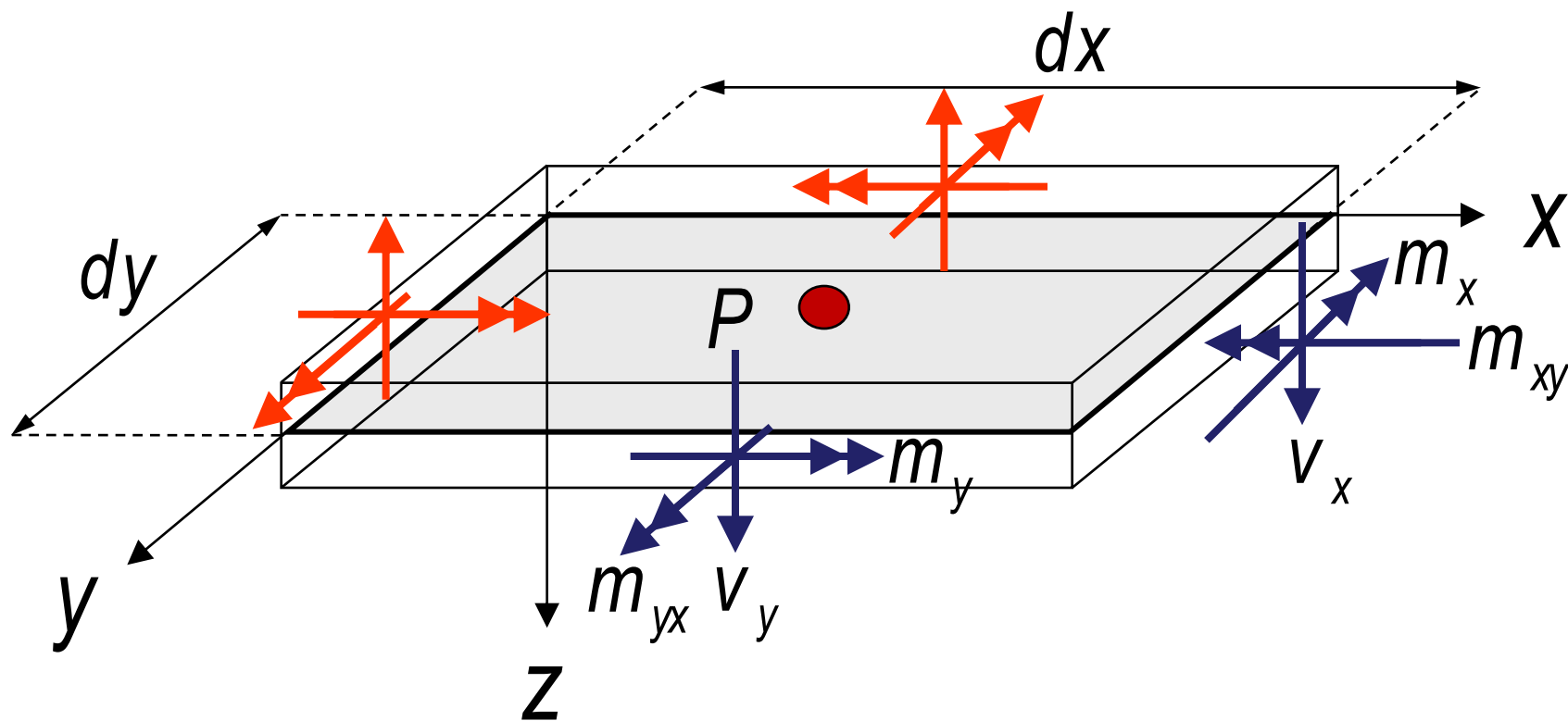
$$L_x / L_y > 2$$

$$L_x / L_y < 0,5$$

$$0,5 \leq L_x / L_y \leq 2$$

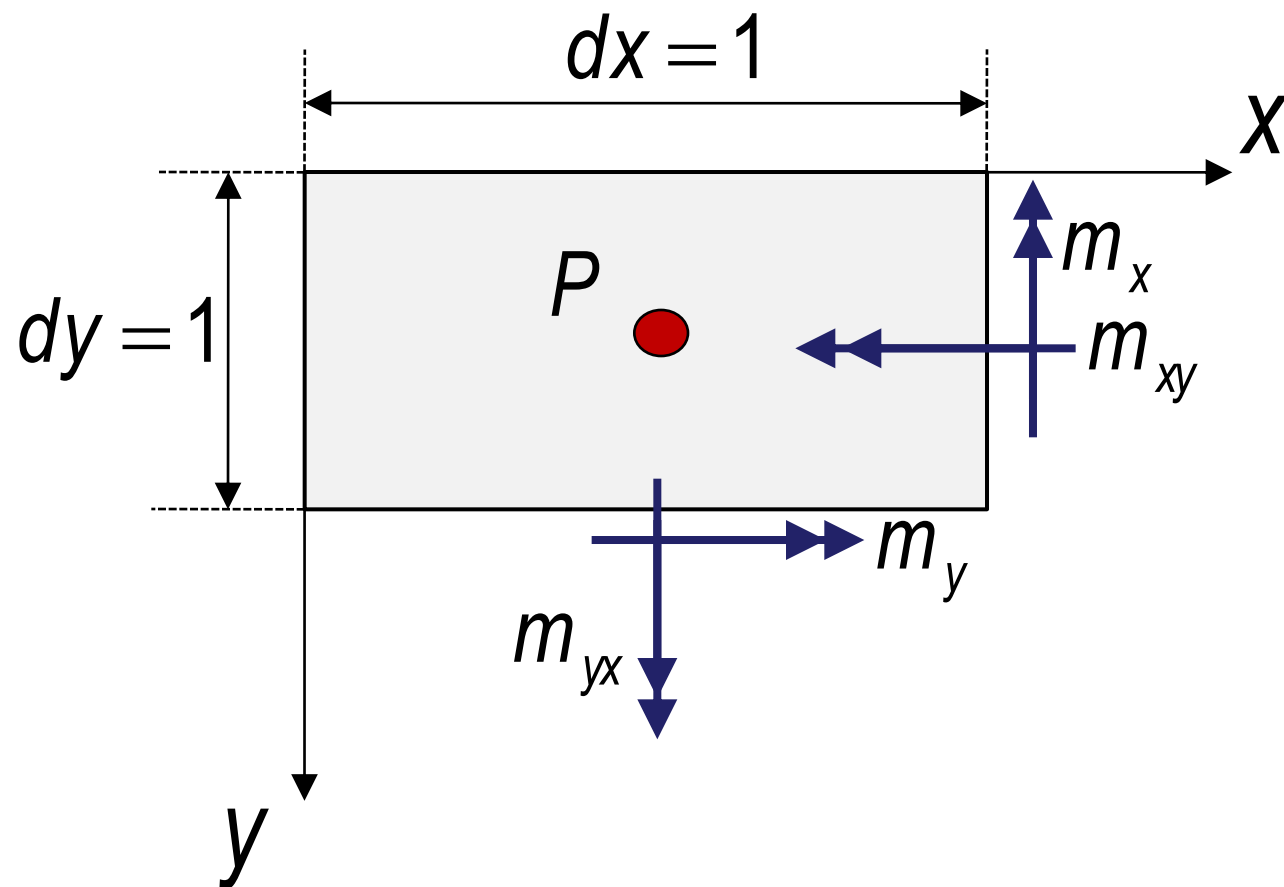
Moments of a two-way slab

Kétirányban teherviselő lemez nyomatókai



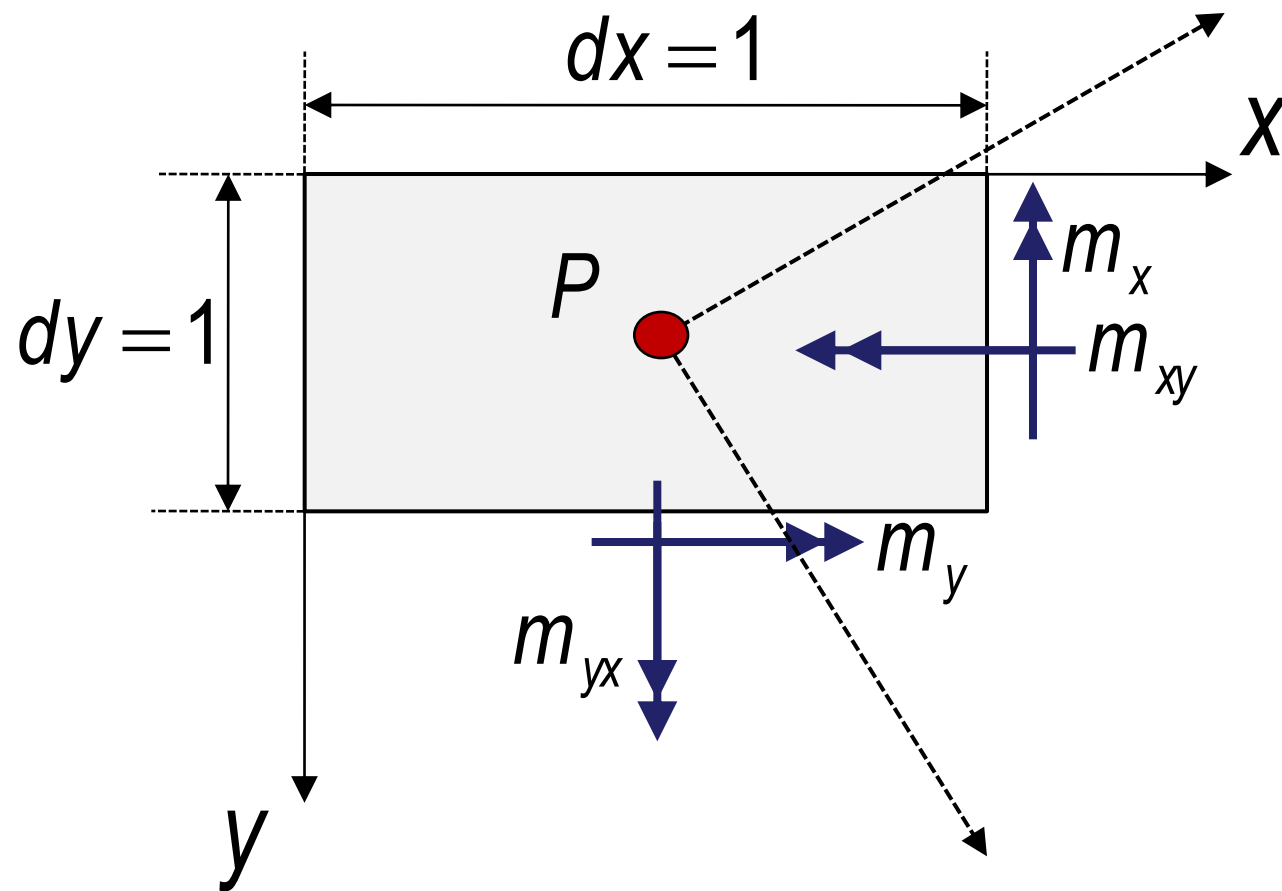
Moments of a two-way slab

Kétirányban teherviselő lemez nyomatékai



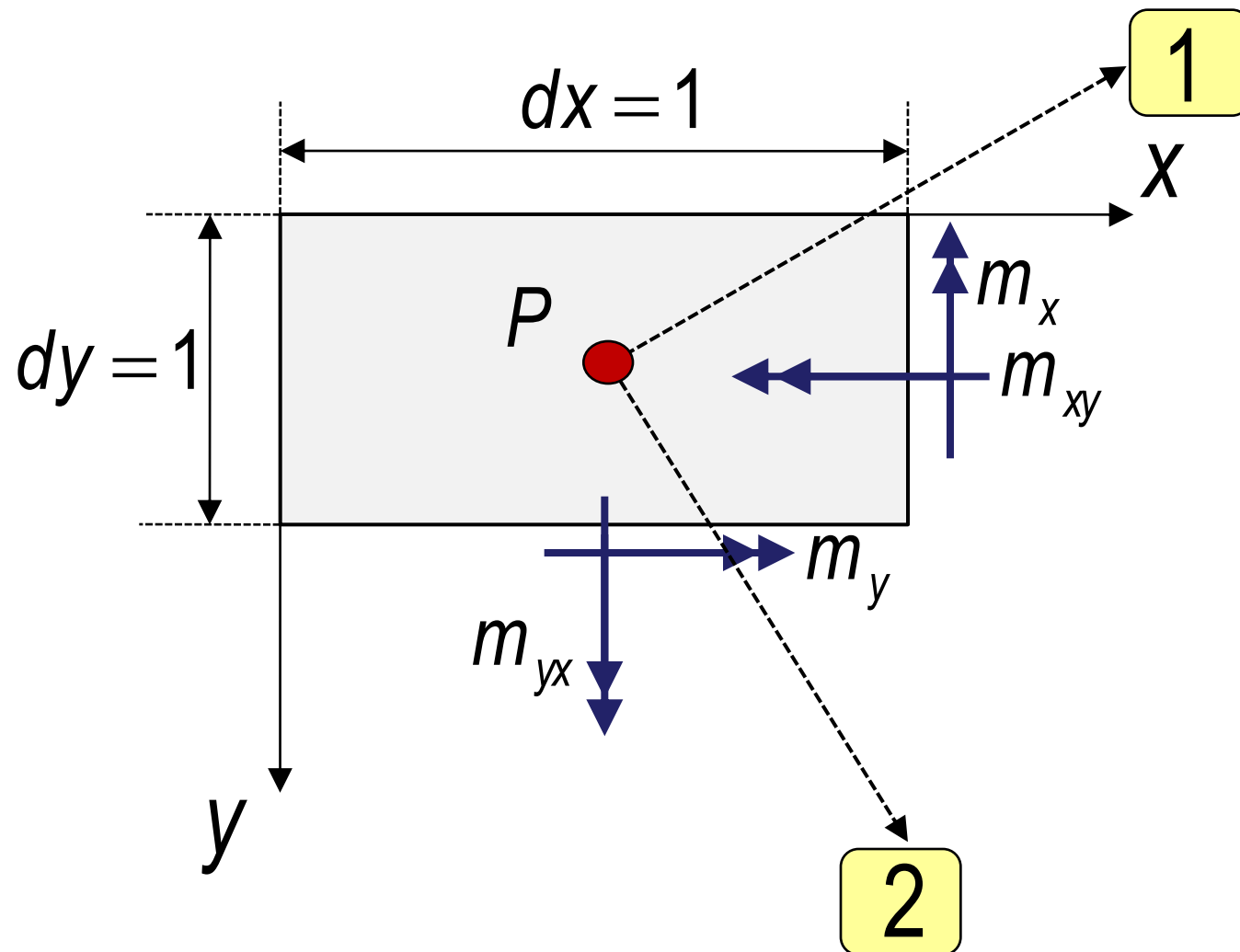
Moments of a two-way slab

Kétirányban teherviselő lemez nyomatékai



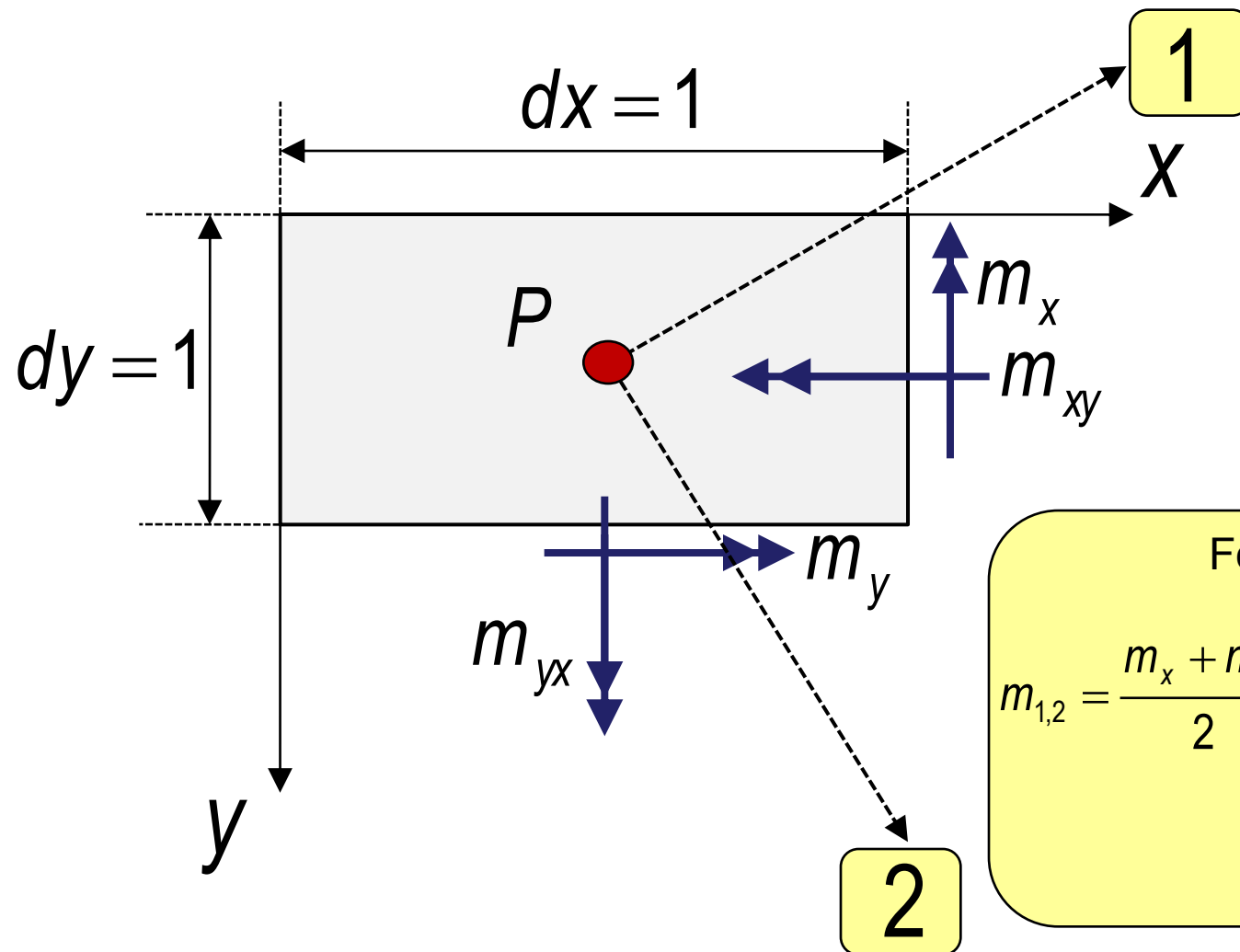
Moments of a two-way slab

Kétirányban teherviselő lemez nyomatékai



Moments of a two-way slab

Kétirányban teherviselő lemez nyomatókai

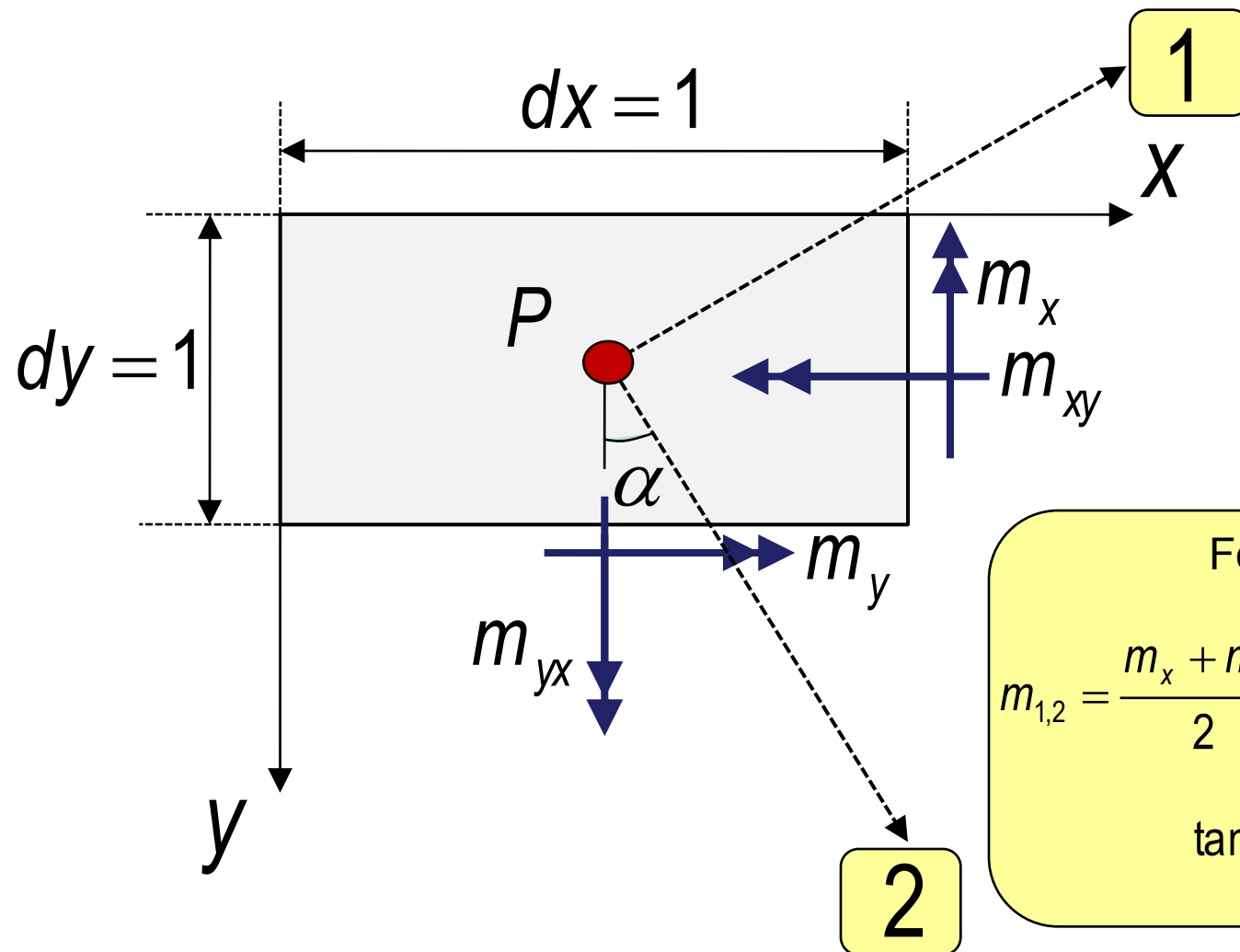


Főnyomatók:

$$m_{1,2} = \frac{m_x + m_y}{2} \pm \sqrt{\frac{(m_x - m_y)^2}{2} + m_{xy}^2}$$

Moments of a two-way slab

Kétirányban teherviselő lemez nyomatókai



Főnyomatók:

$$m_{1,2} = \frac{m_x + m_y}{2} \pm \sqrt{\frac{(m_x - m_y)^2}{2} + m_{xy}^2}$$

$$\tan 2\alpha = \frac{2 \cdot m_{xy}}{m_y - m_x}$$

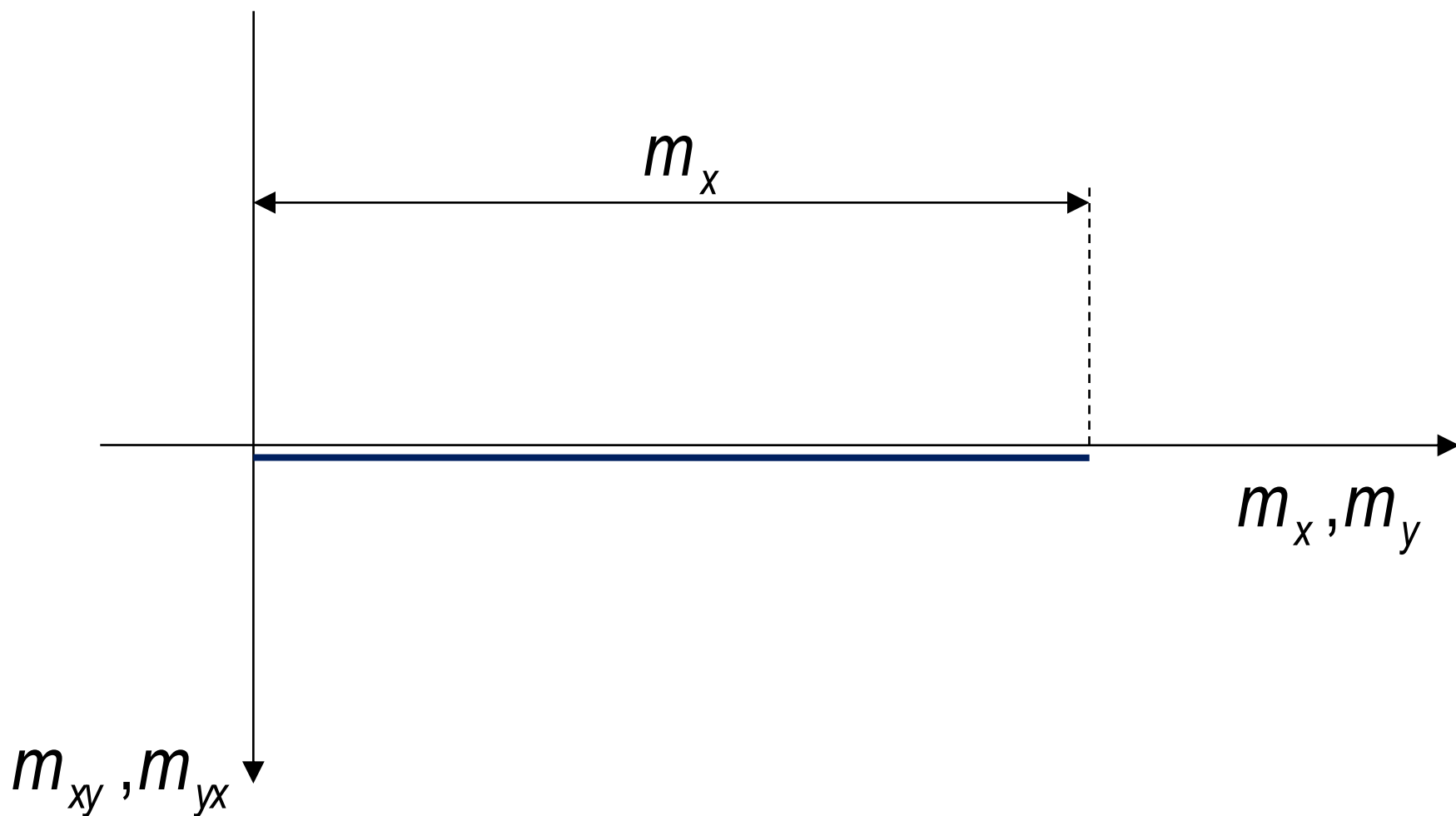
Moments of a two-way slab

Kétirányban teherviselő lemez nyomatékai



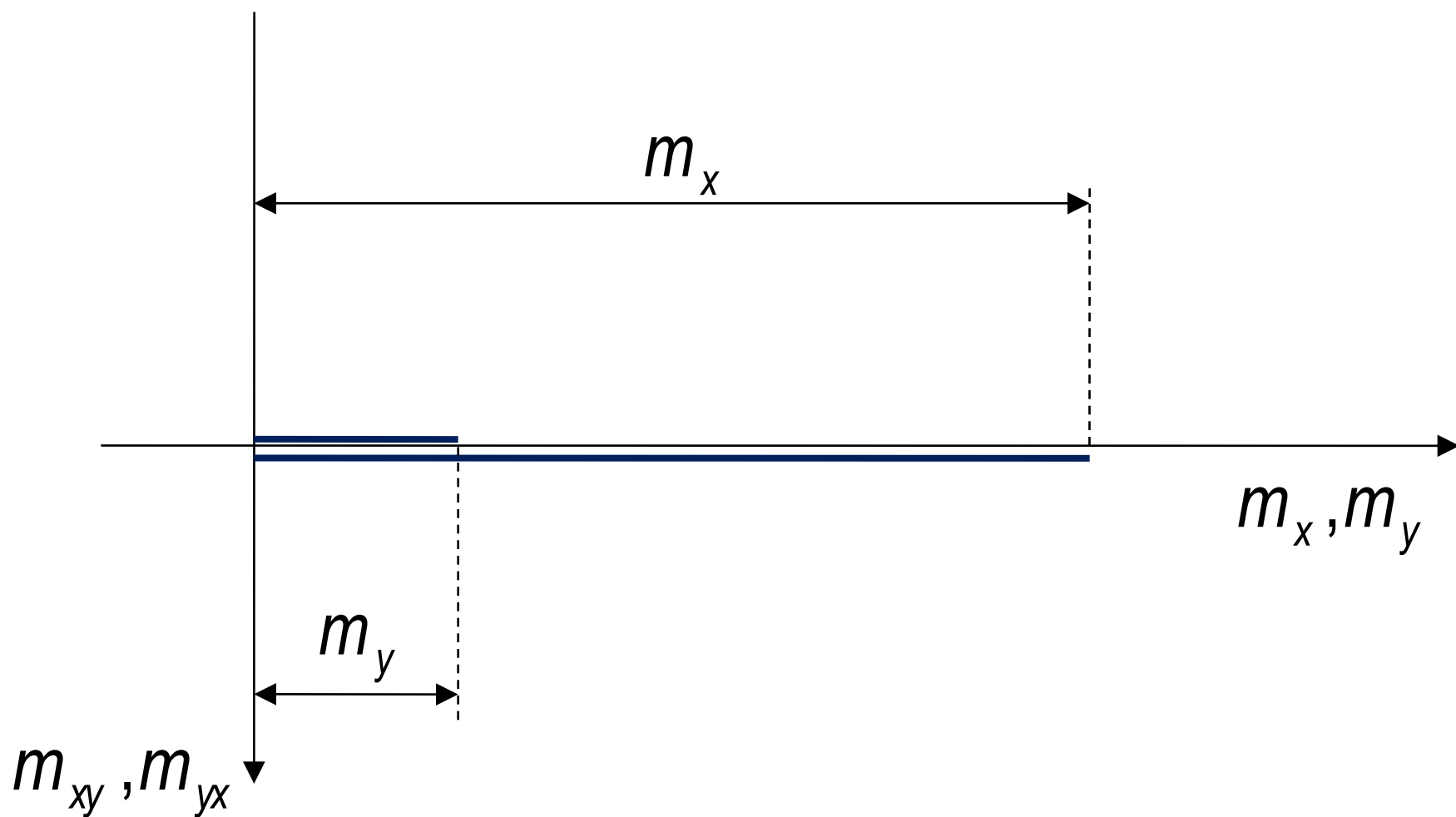
Moments of a two-way slab

Kétirányban teherviselő lemez nyomatékai



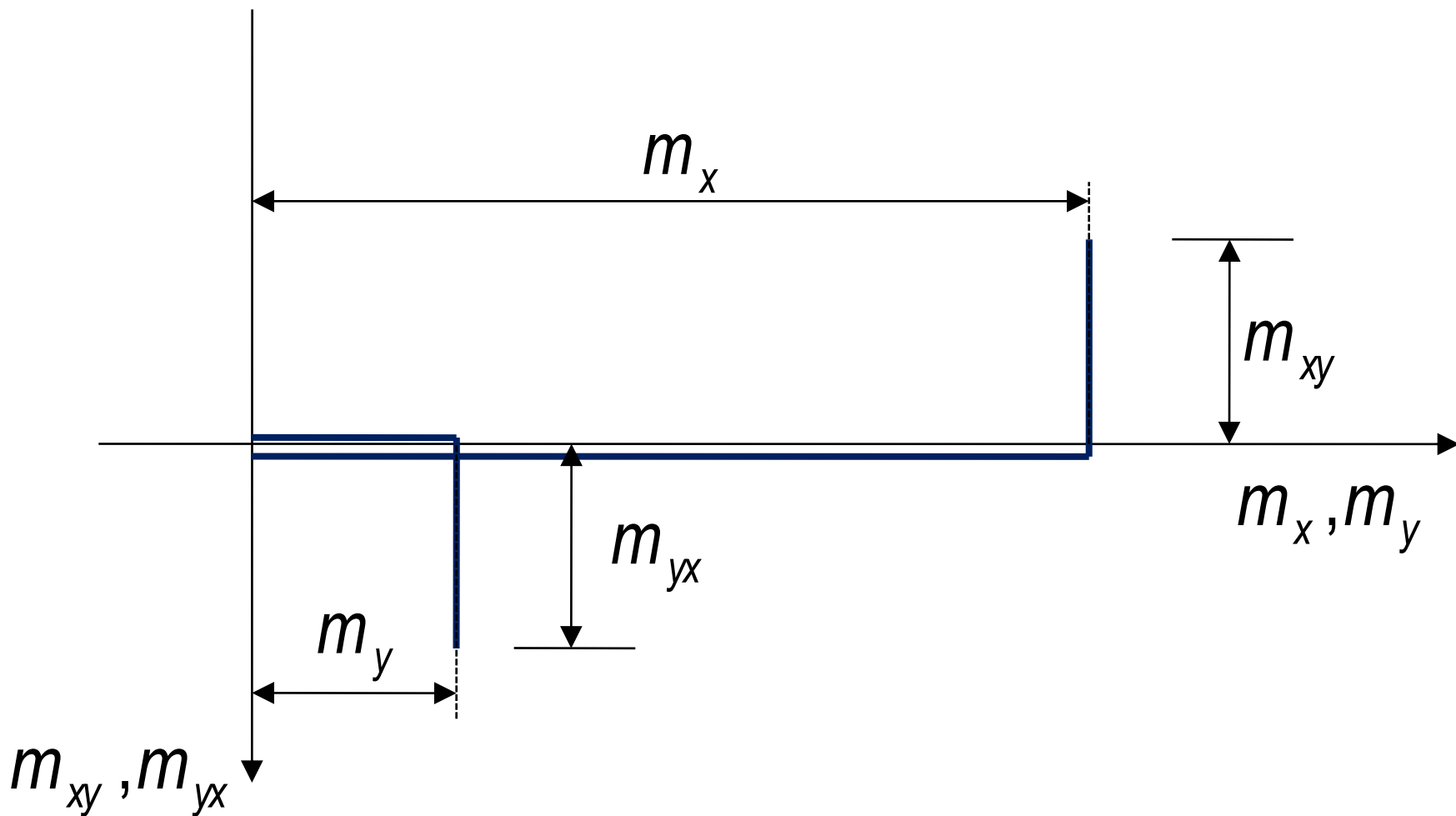
Moments of a two-way slab

Kétirányban teherviselő lemez nyomatékai



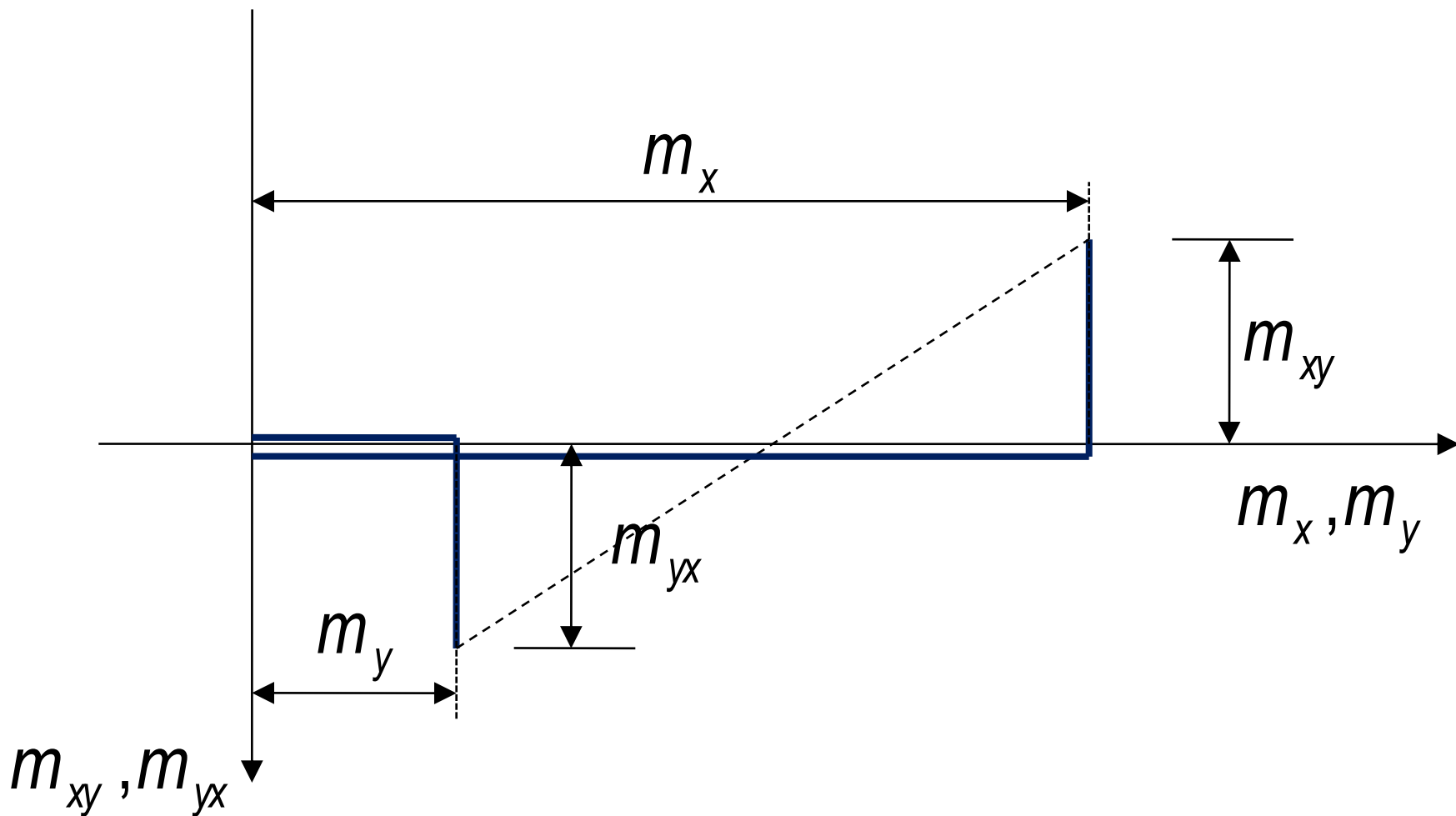
Moments of a two-way slab

Kétirányban teherviselő lemez nyomatókai



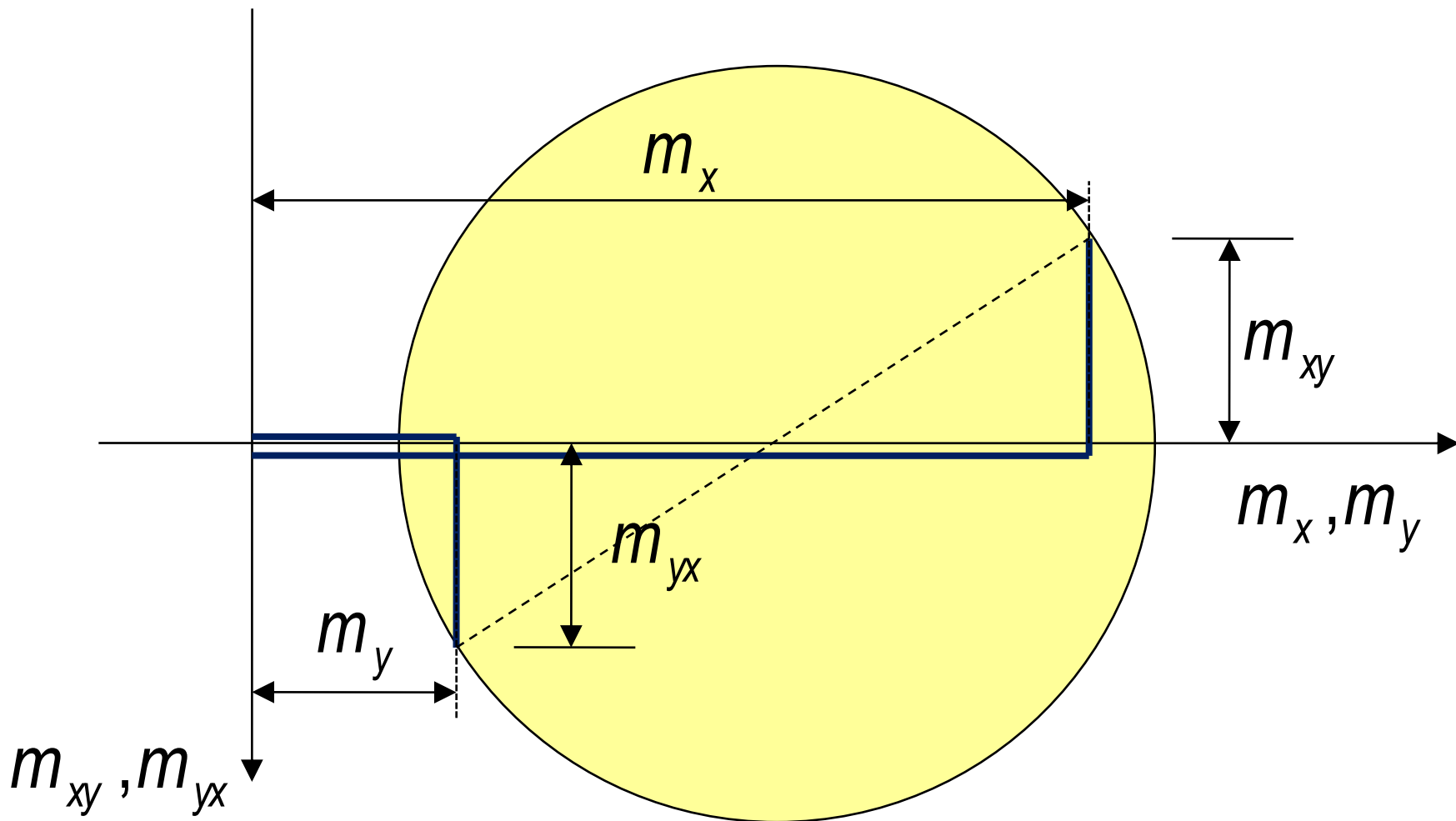
Moments of a two-way slab

Kétirányban teherviselő lemez nyomatókai



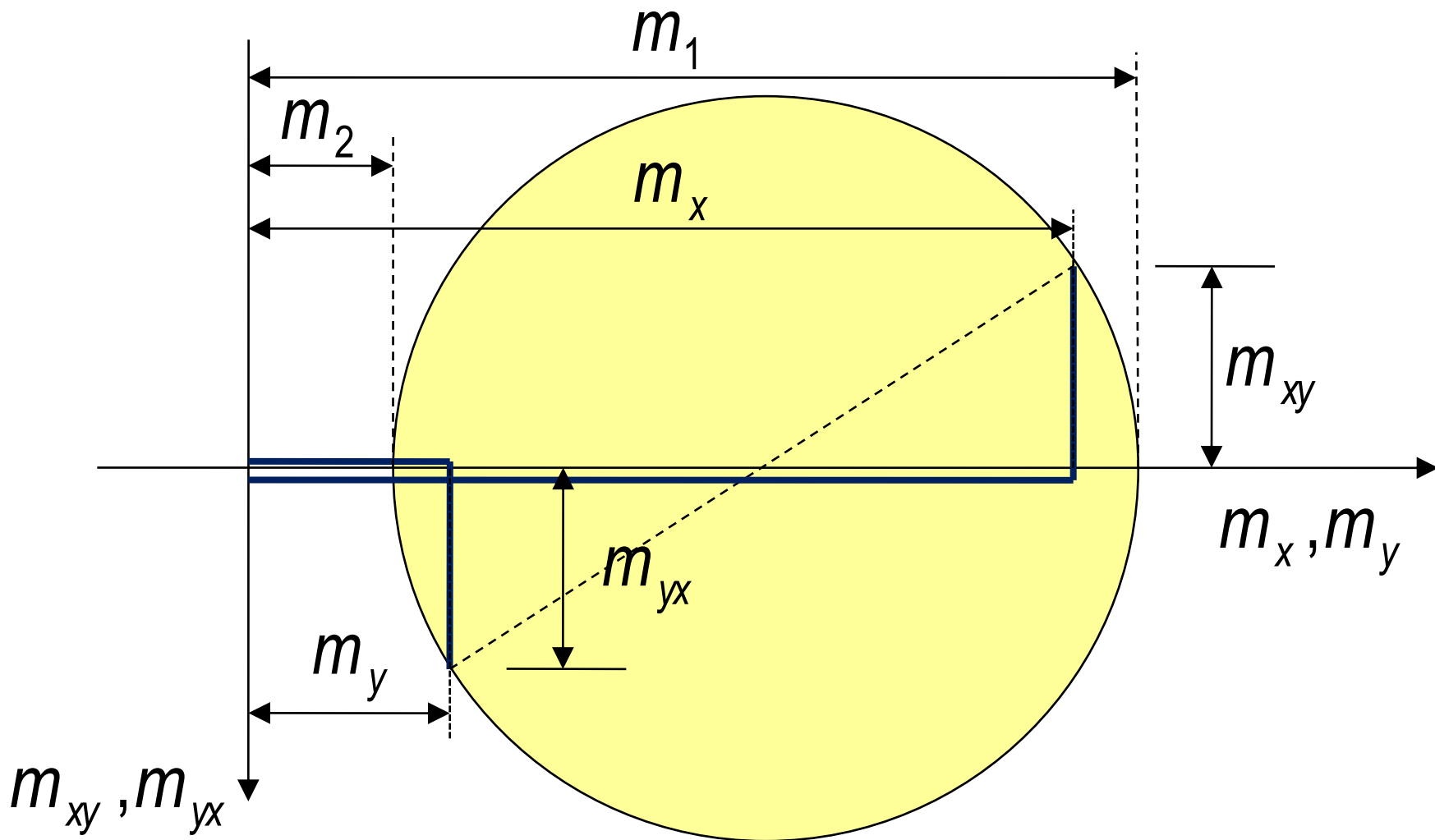
Moments of a two-way slab

Kétirányban teherviselő lemez nyomatókai



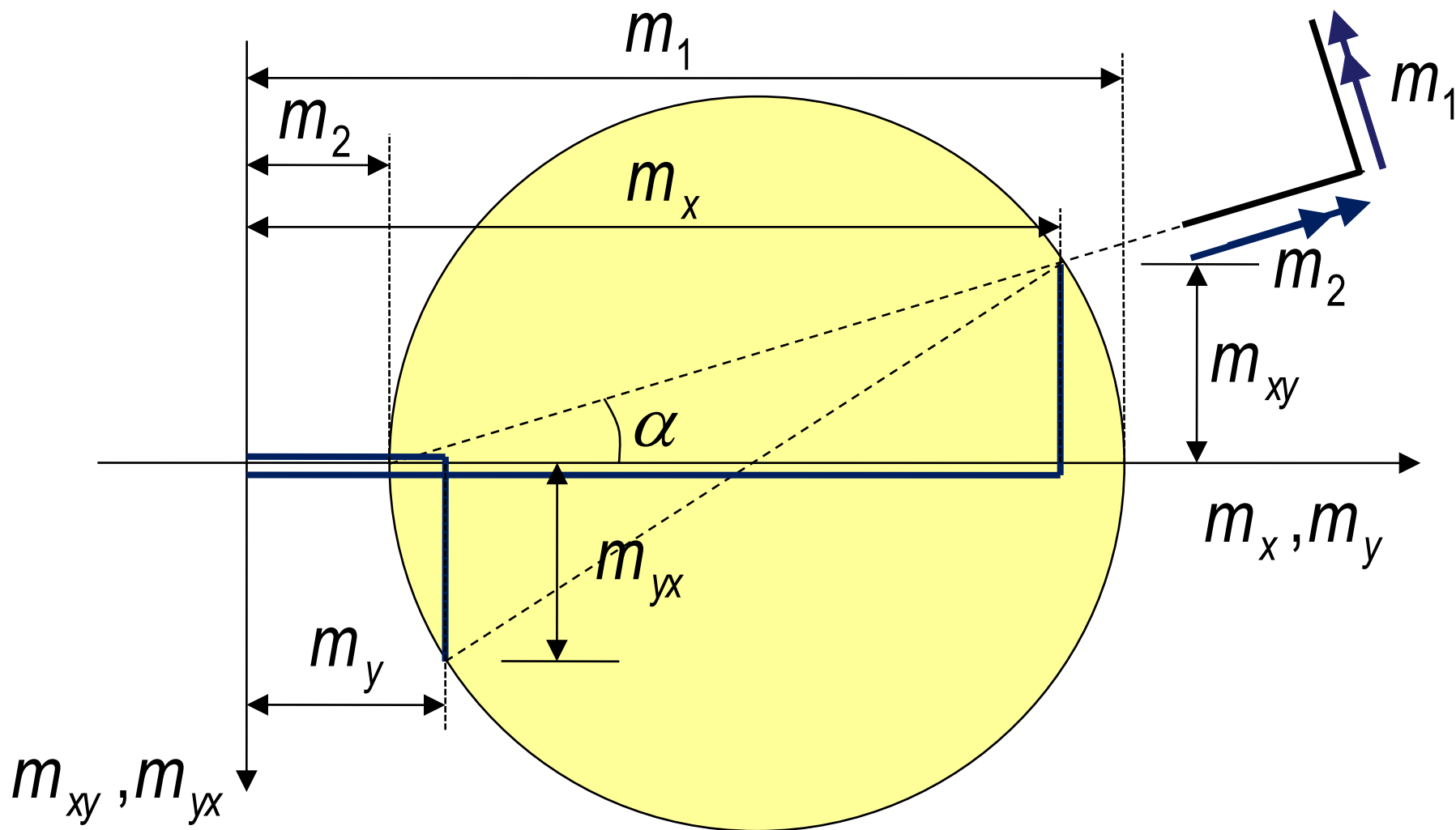
Moments of a two-way slab

Kétirányban teherviselő lemez nyomatókai



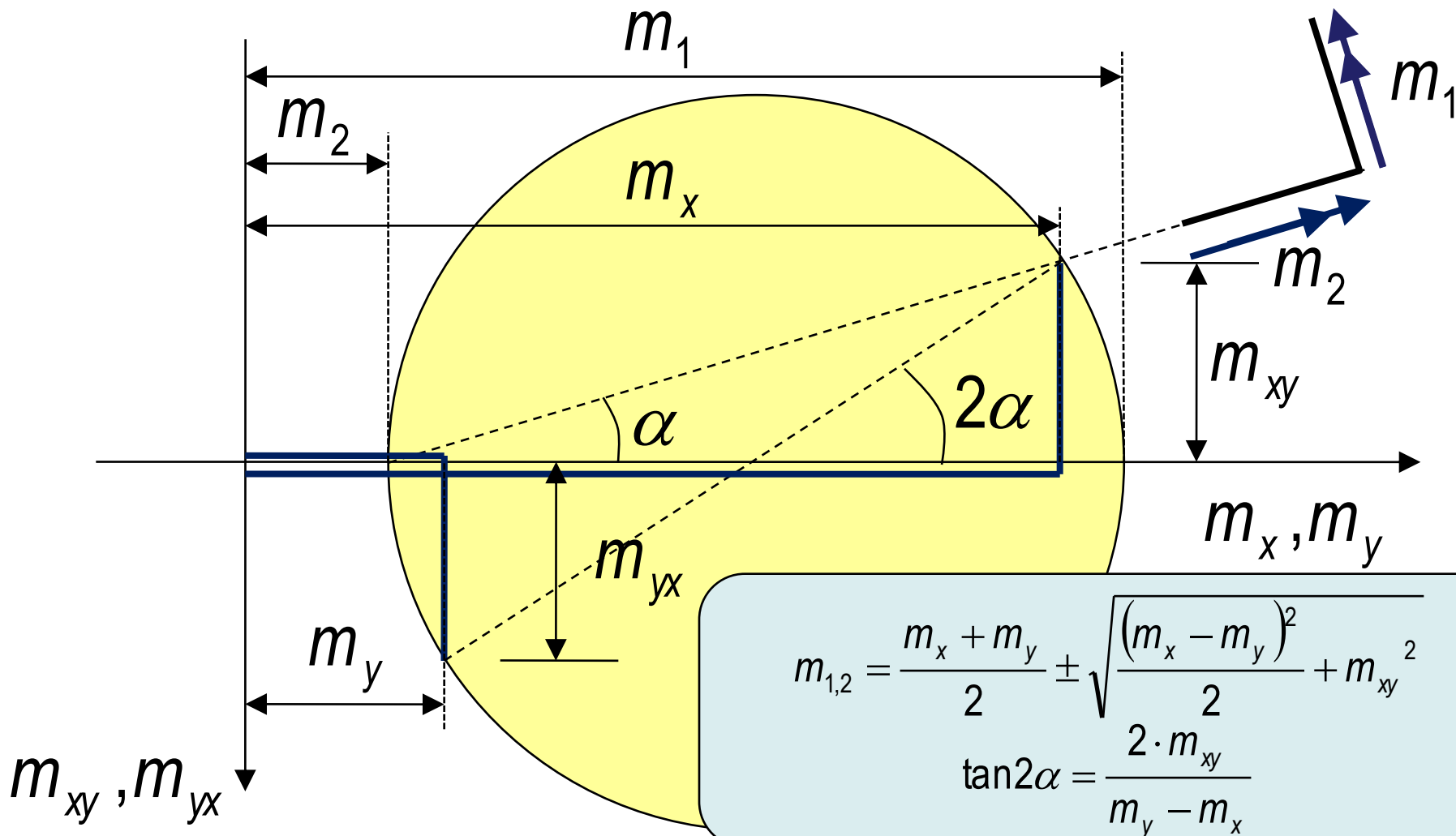
Moments of a two-way slab

Kétirányban teherviselő lemez nyomatókai



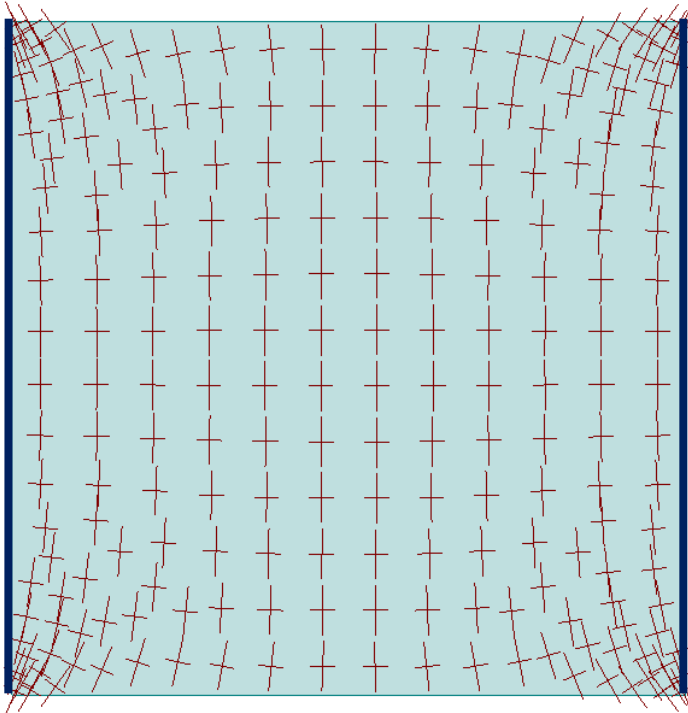
Moments of a two-way slab

Kétirányban teherviselő lemez nyomatókai



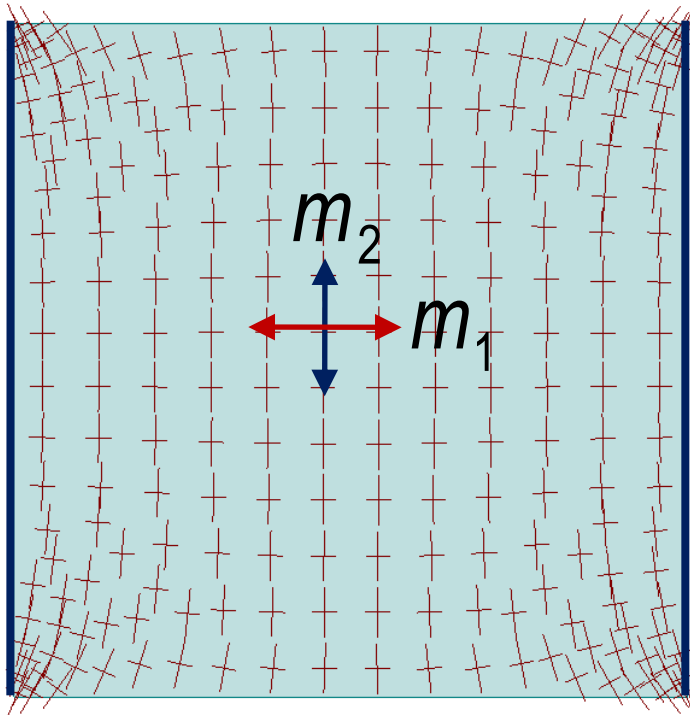
Moments of one-way and two-way slabs

Egy- és kétirányban teherviselő lemez nyomatókai



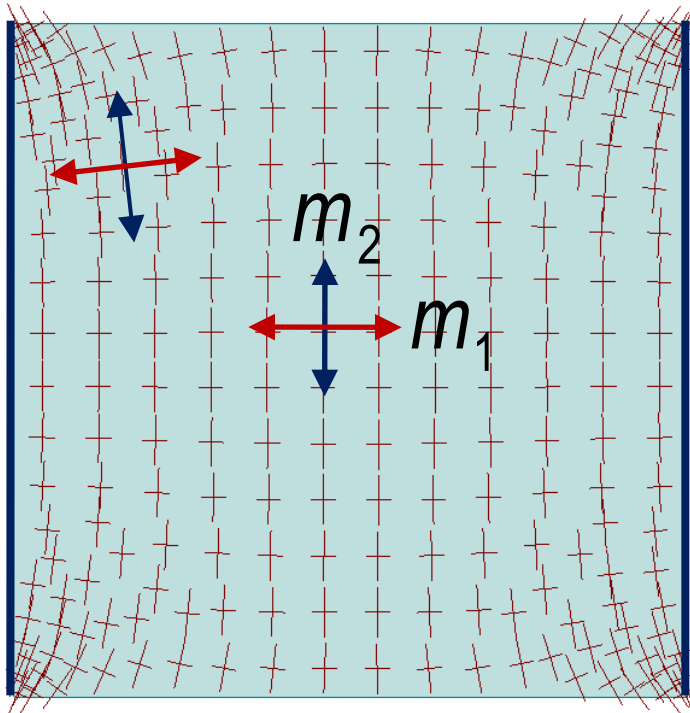
Moments of one-way and two-way slabs

Egy- és kétirányban teherviselő lemez nyomatékai



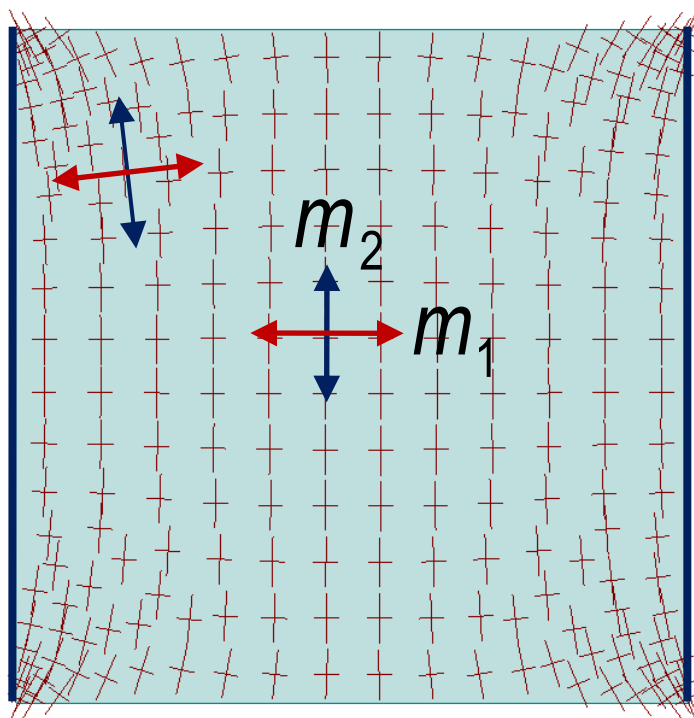
Moments of one-way and two-way slabs

Egy- és kétirányban teherviselő lemez nyomatékai



Moments of one-way and two-way slabs

Egy- és kétirányban teherviselő lemez nyomatókai

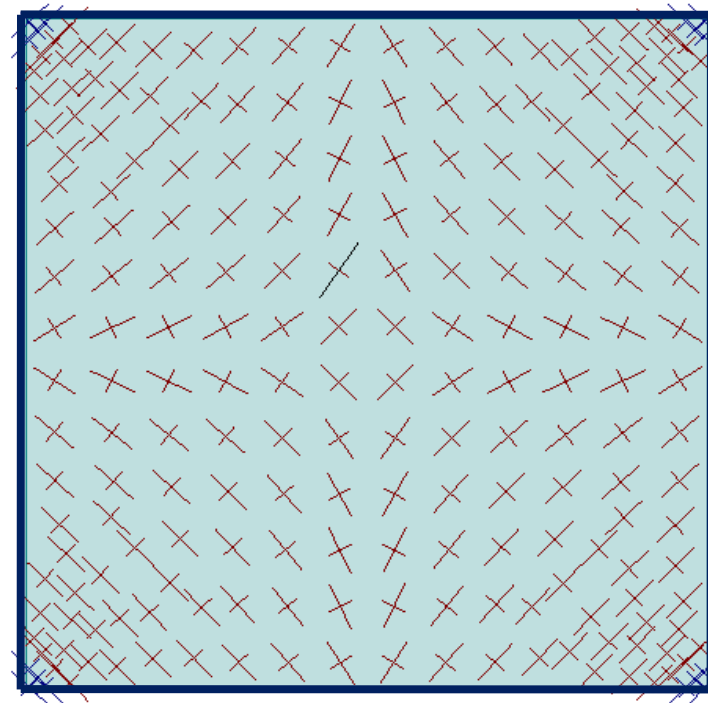
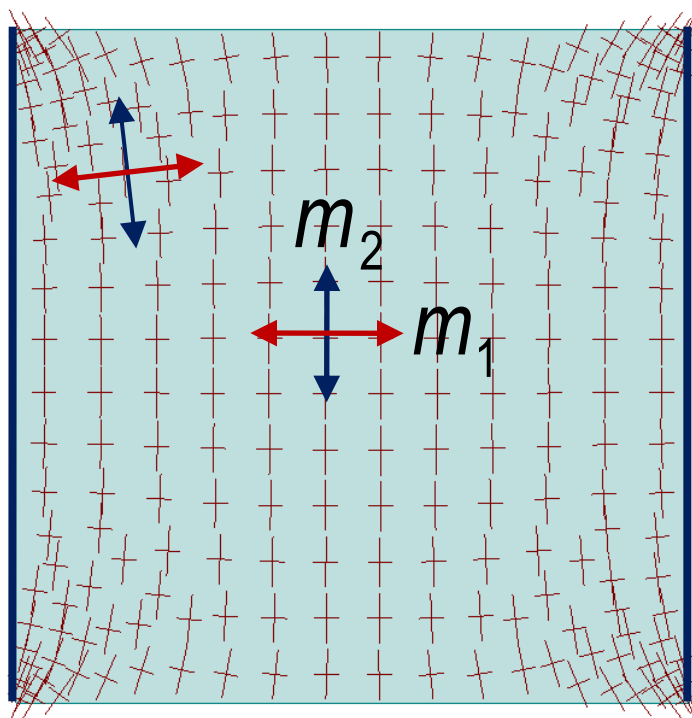


$$m_1 = m_x$$

$$m_2 = m_y = \mu_c \cdot m_x$$

Moments of one-way and two-way slabs

Egy- és kétirányban teherviselő lemez nyomatókai

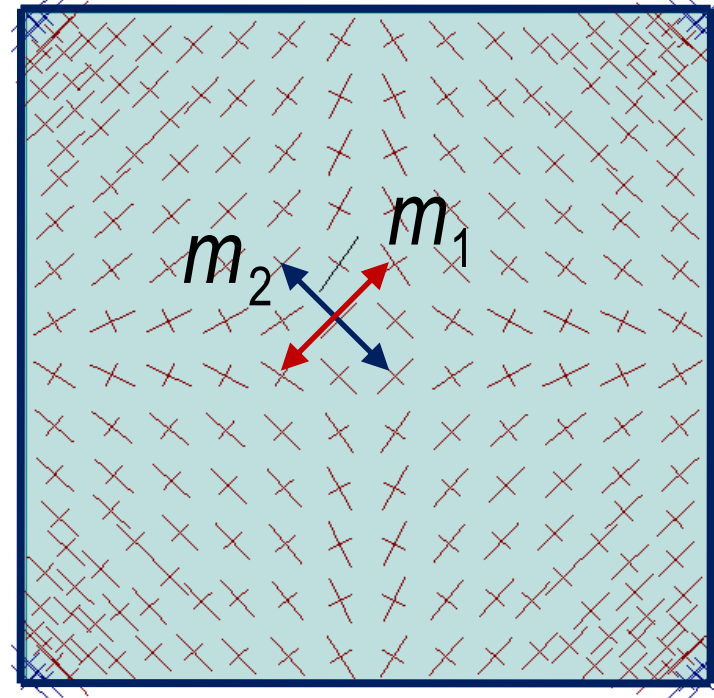
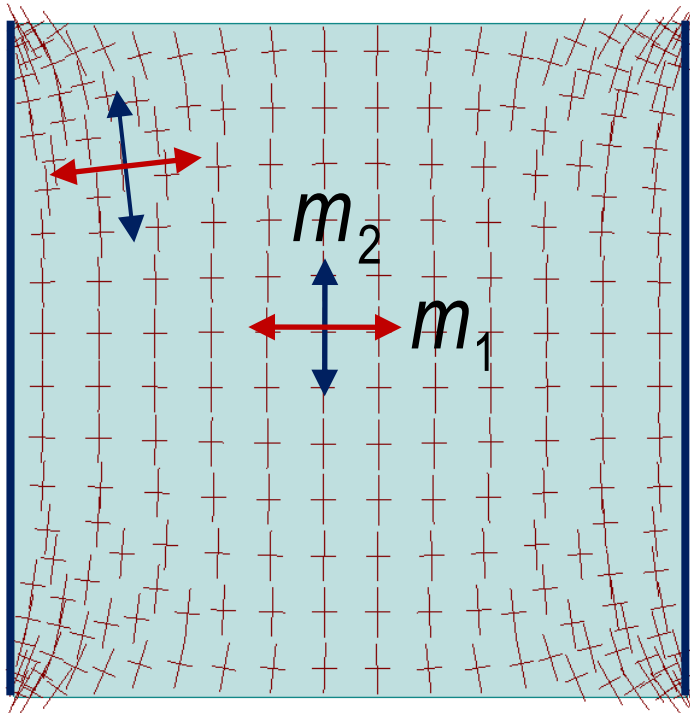


$$m_1 = m_x$$

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Moments of one-way and two-way slabs

Egy- és kétirányban teherviselő lemez nyomatókai

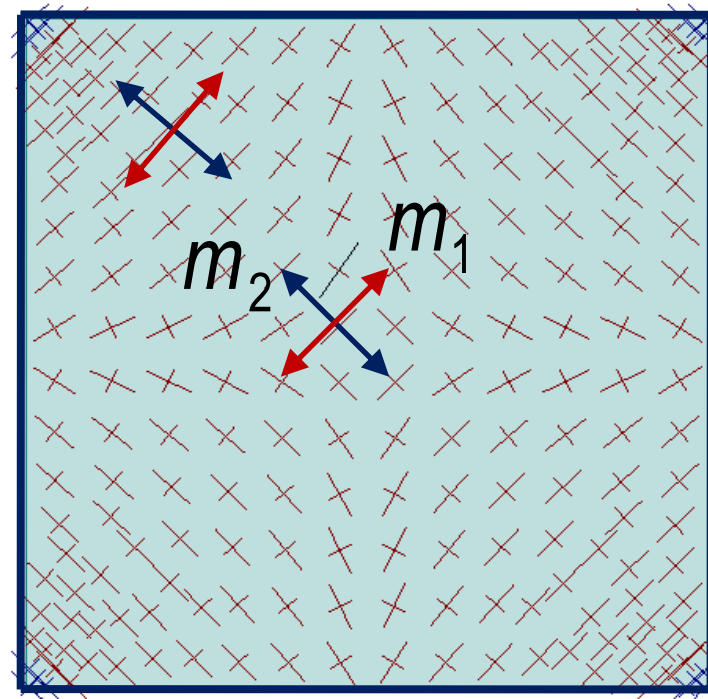
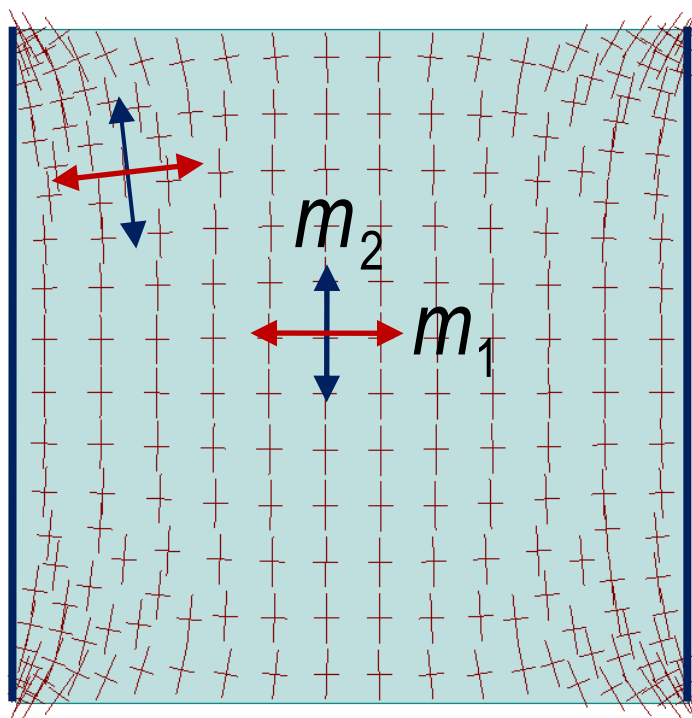


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Moments of one-way and two-way slabs

Egy- és kétirányban teherviselő lemez nyomatókai

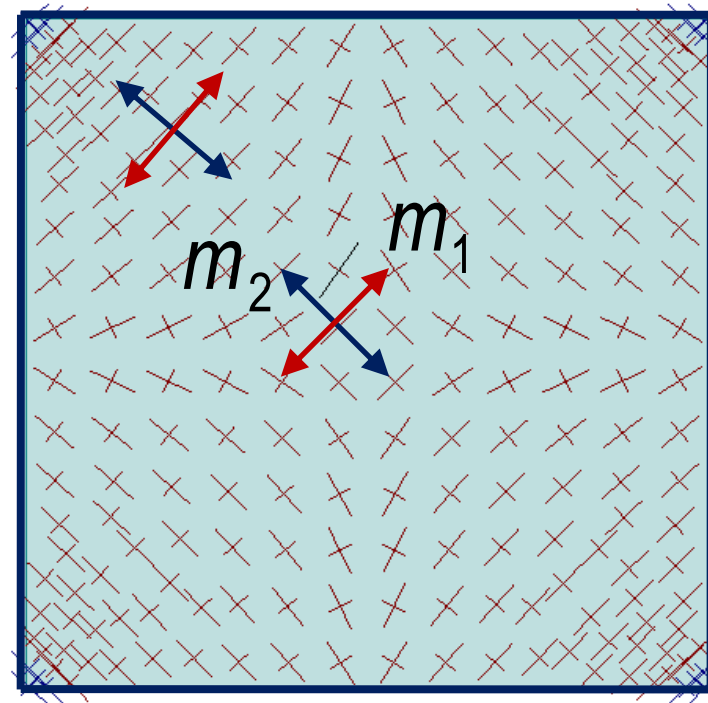
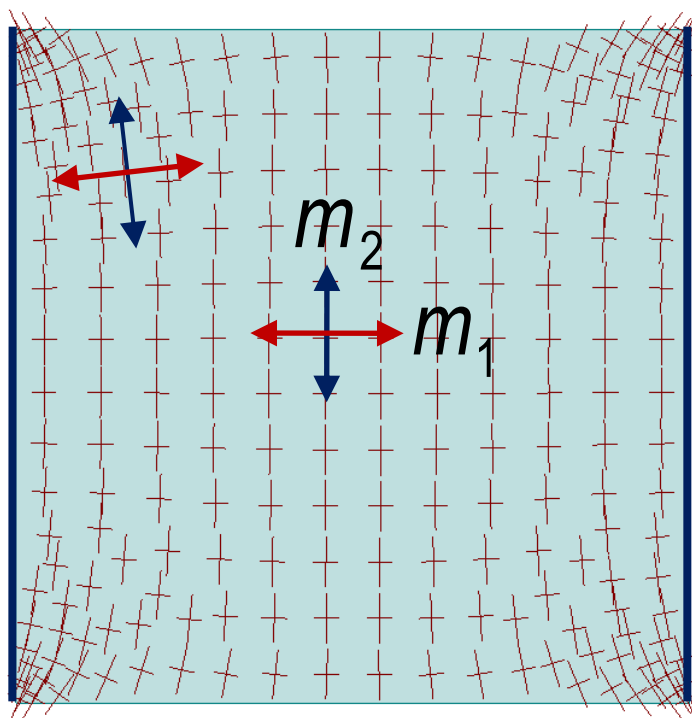


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Moments of one-way and two-way slabs

Egy- és kétirányban teherviselő lemez nyomatókai



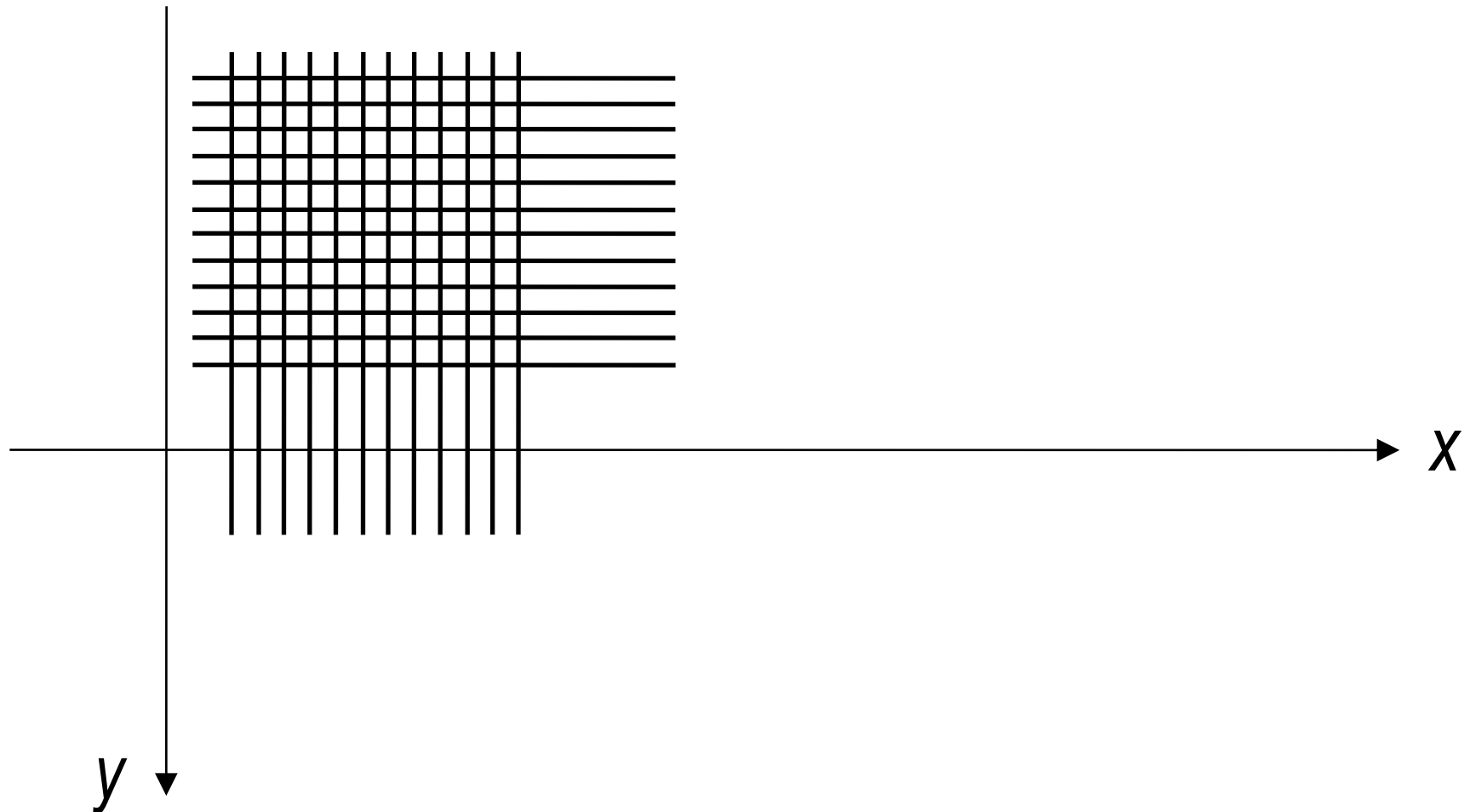
$$m_1 = m_x$$

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$$m_{1,2} = \frac{m_x + m_y}{2} \pm \sqrt{\frac{(m_x - m_y)^2}{2} + m_{xy}^2}$$

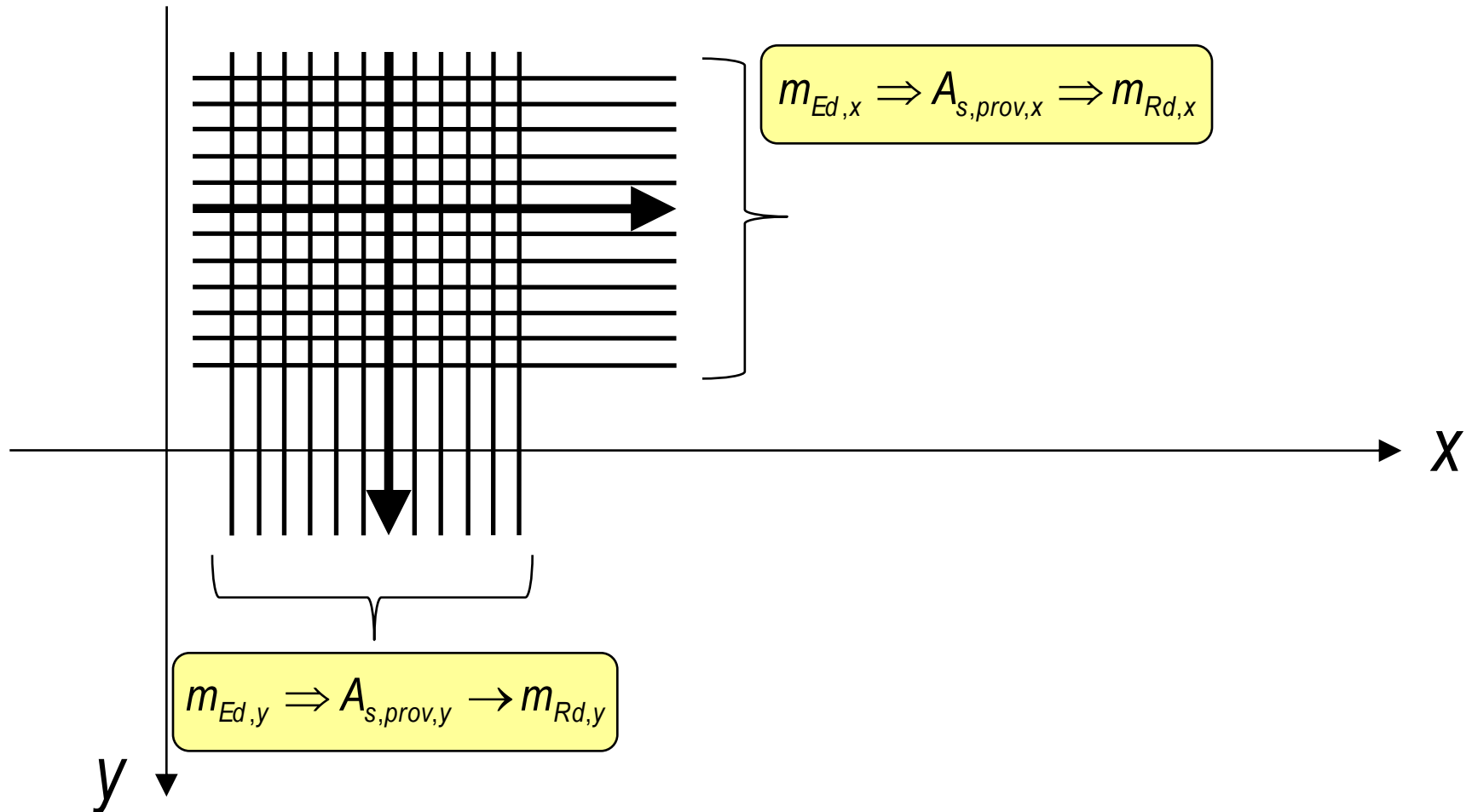
Ultimate moment of a two-way slab in α direction

Kétirányban vasalt lemez α irányú törőnyomatéka



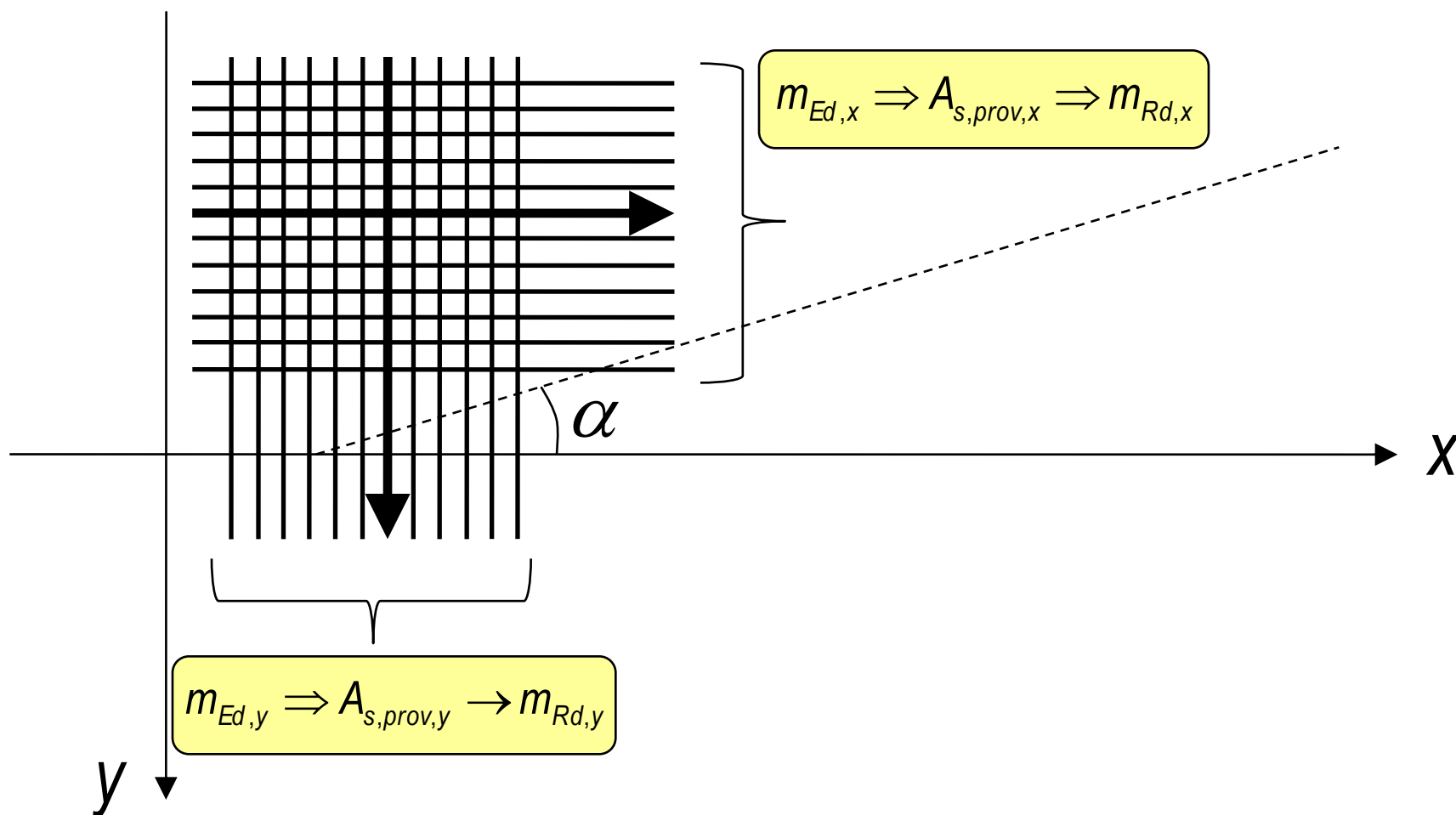
Ultimate moment of a two-way slab in α direction

Kétirányban vasalt lemez α irányú törőnyomatéka



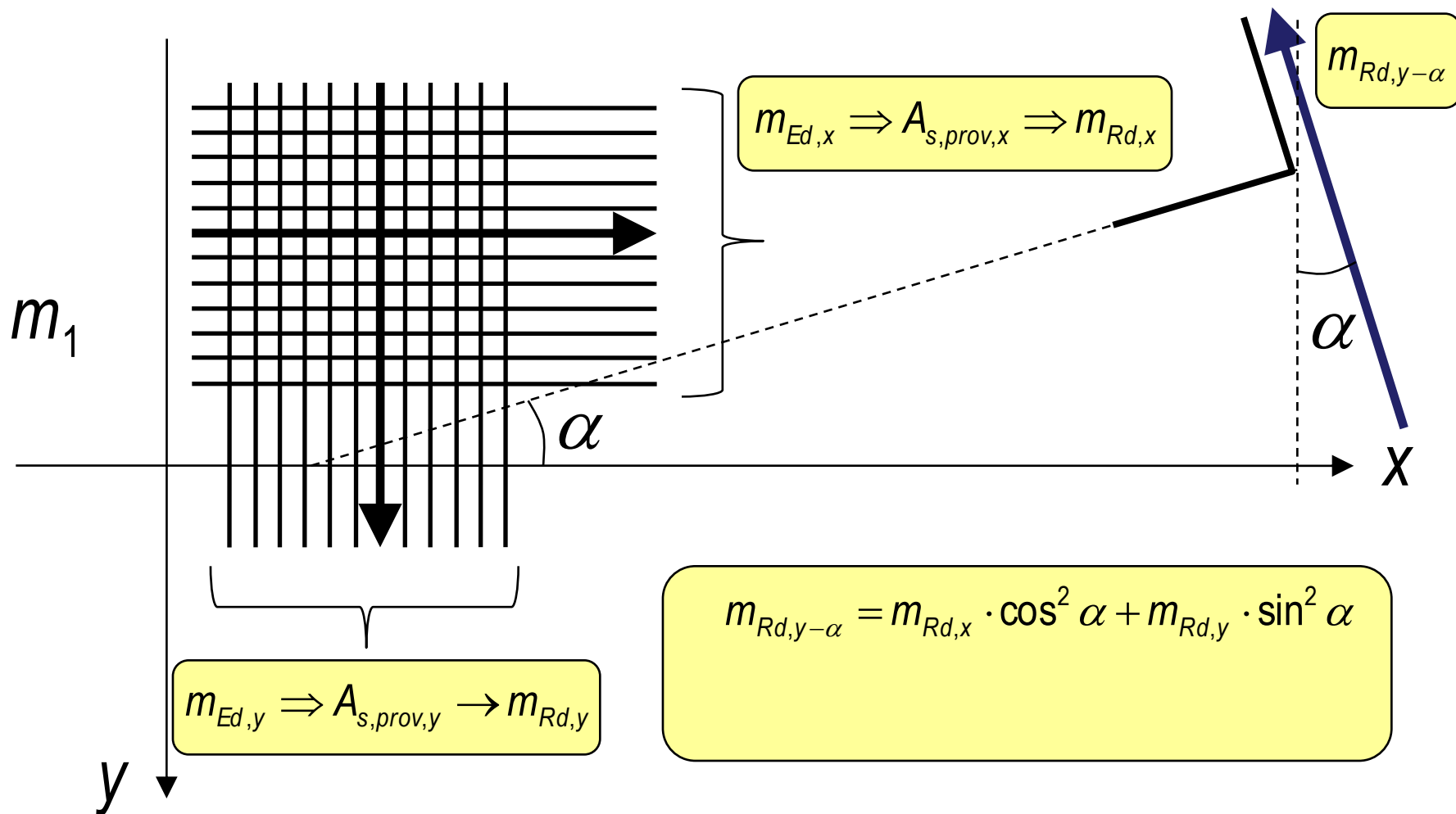
Ultimate moment of a two-way slab in α direction

Kétirányban vasalt lemez α irányú törőnyomatéka



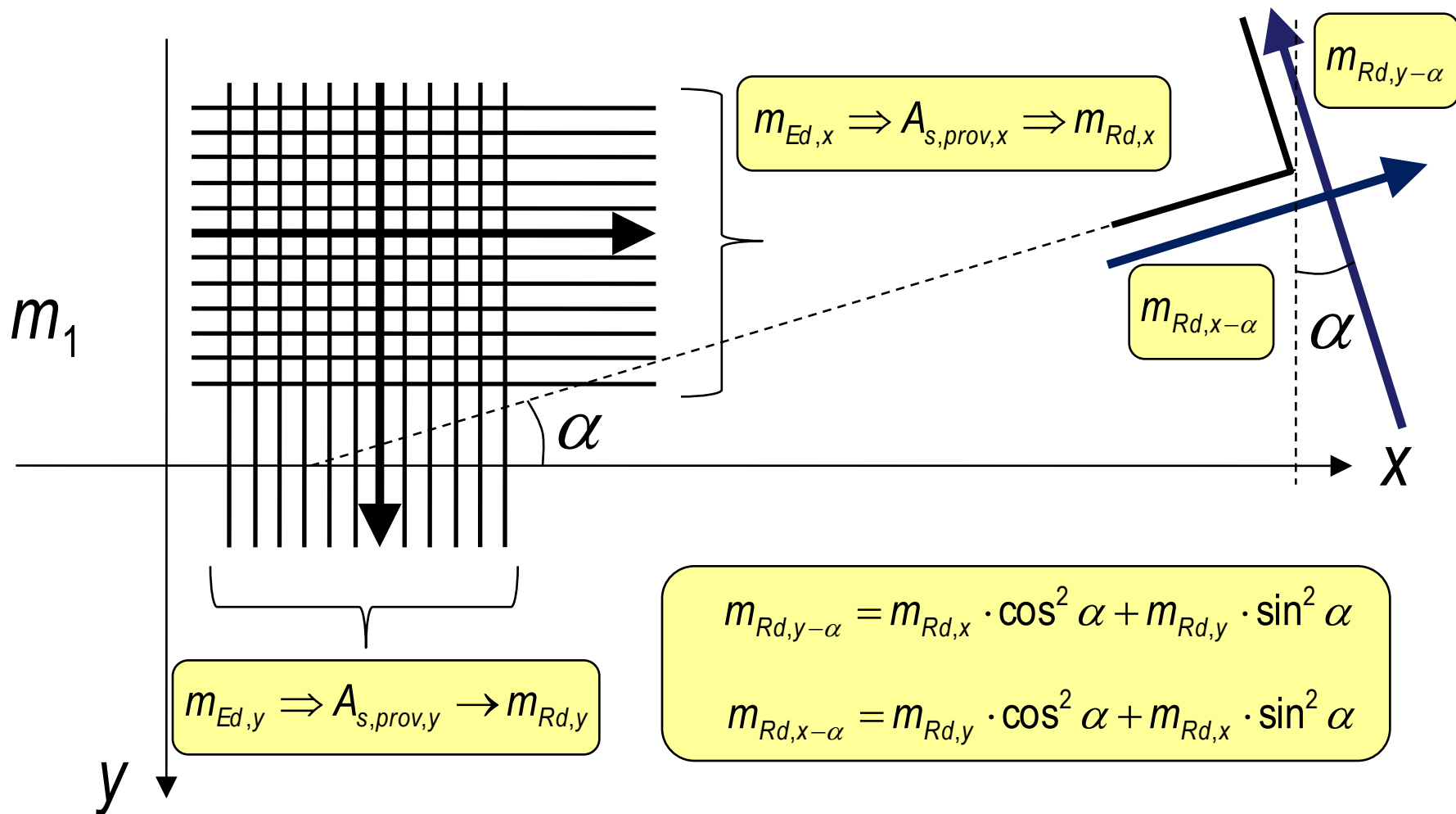
Ultimate moment of a two-way slab in α direction

Kétirányban vasalt lemez α irányú törőnyomatéka



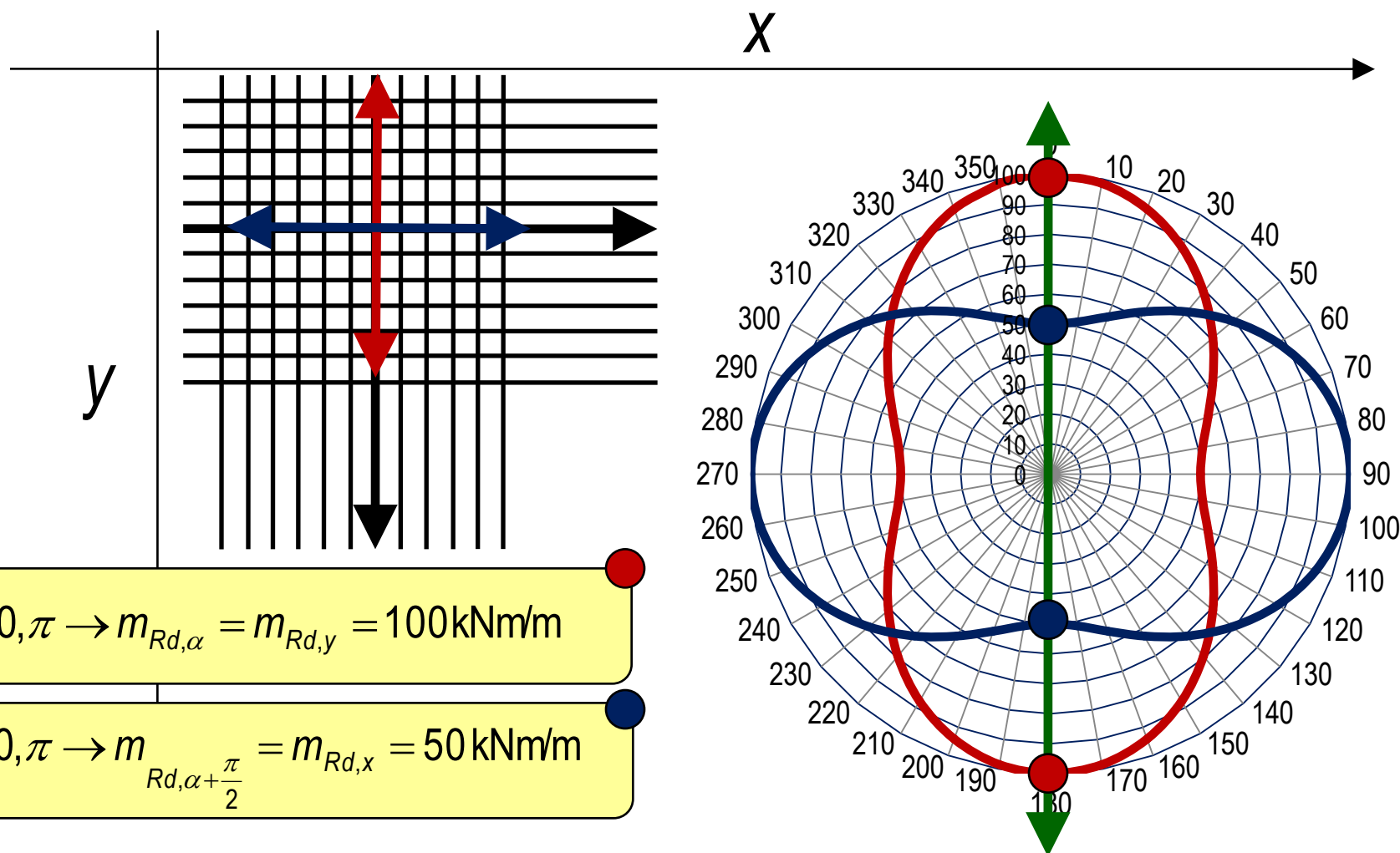
Ultimate moment of a two-way slab in α direction

Kétirányban vasalt lemez α irányú törőnyomatéka



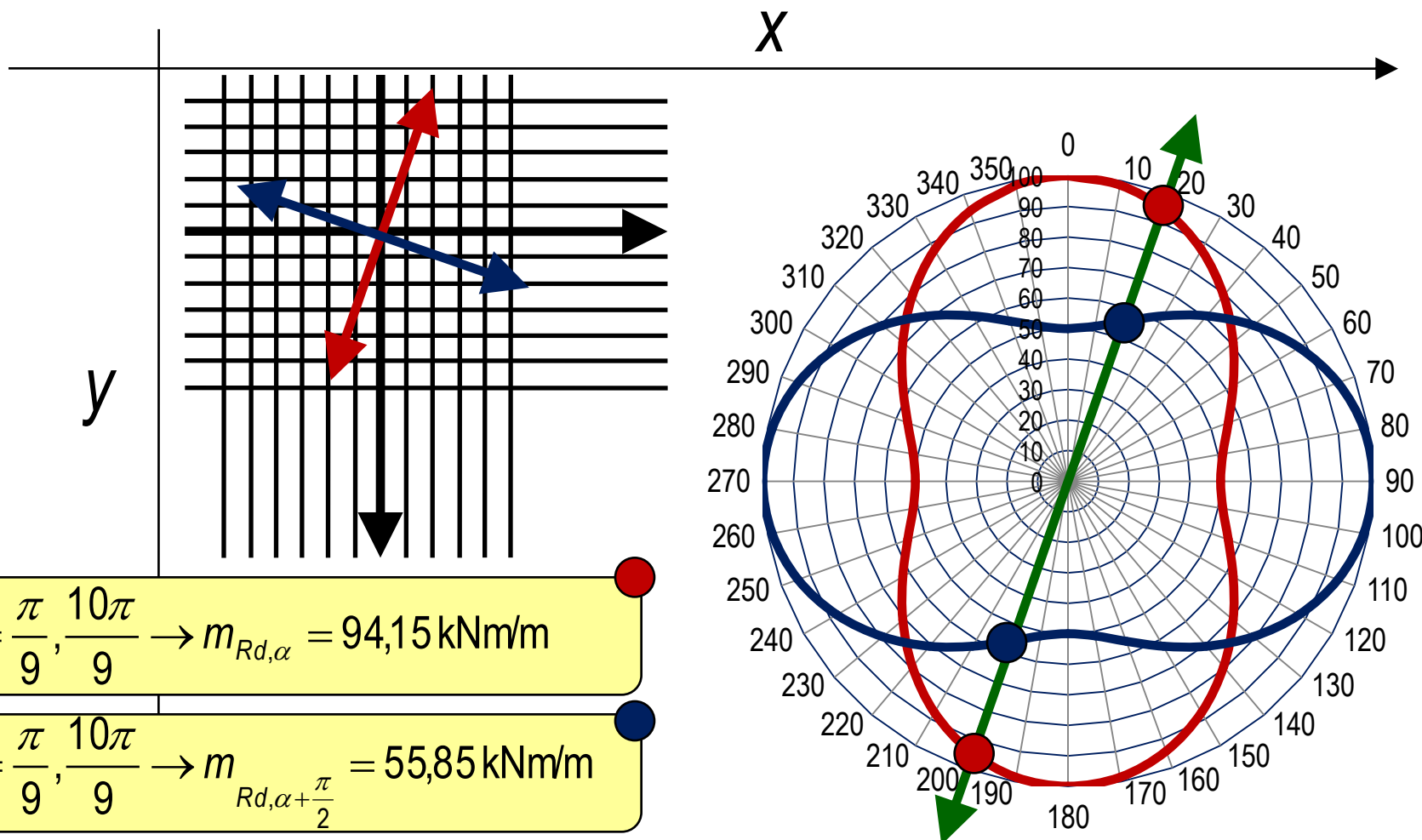
Ultimate moment of a two-way slab in α direction

Kétirányban vasalt lemez α irányú törőnyomatéka



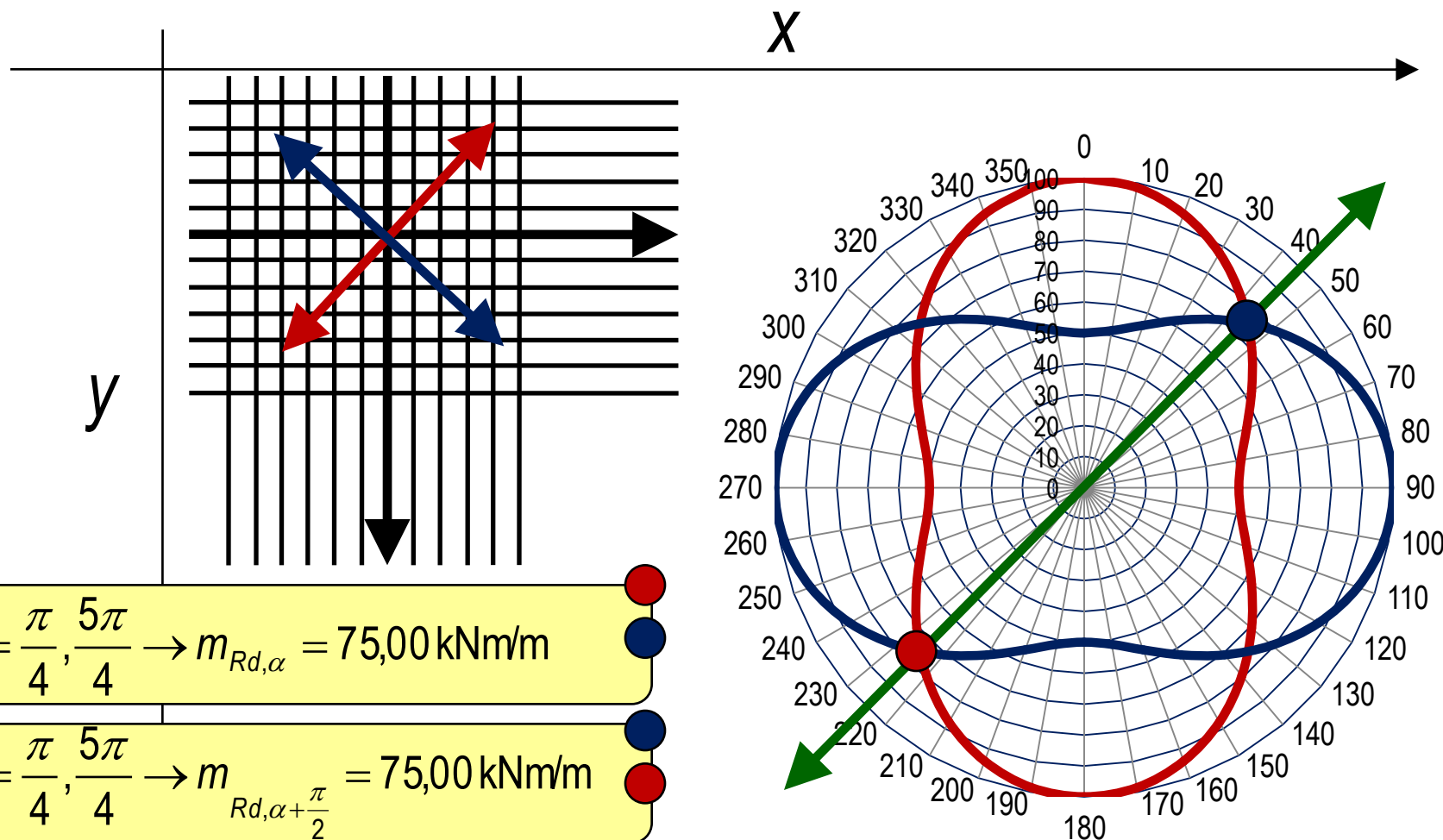
Ultimate moment of a two-way slab in α direction

Kétirányban vasalt lemez α irányú törőnyomatéka



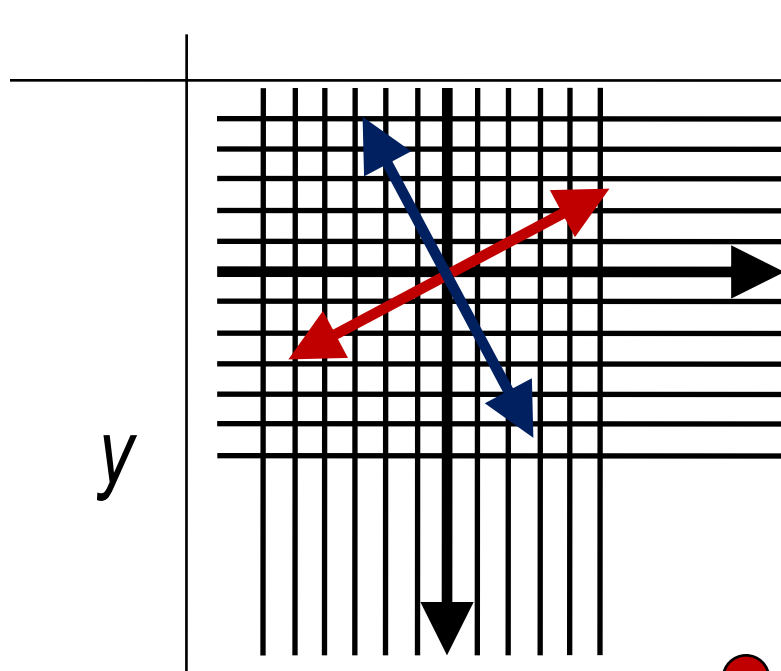
Ultimate moment of a two-way slab in α direction

Kétirányban vasalt lemez α irányú törőnyomatéka



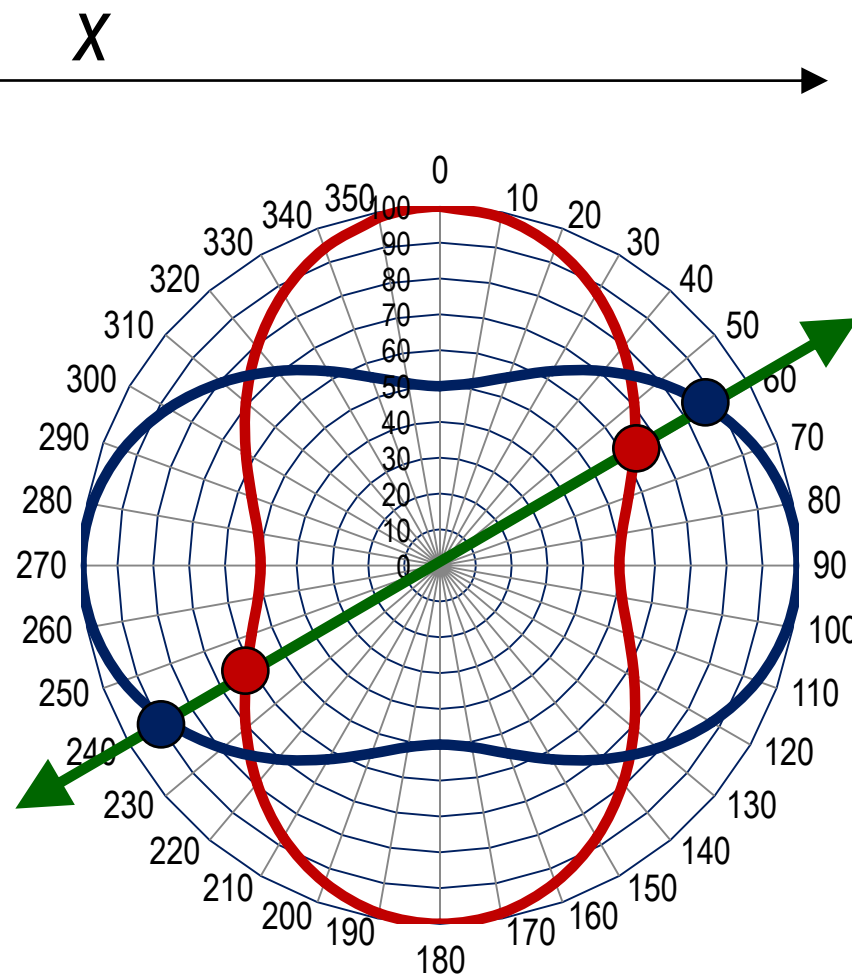
Ultimate moment of a two-way slab in α direction

Kétirányban vasalt lemez α irányú törőnyomatéka



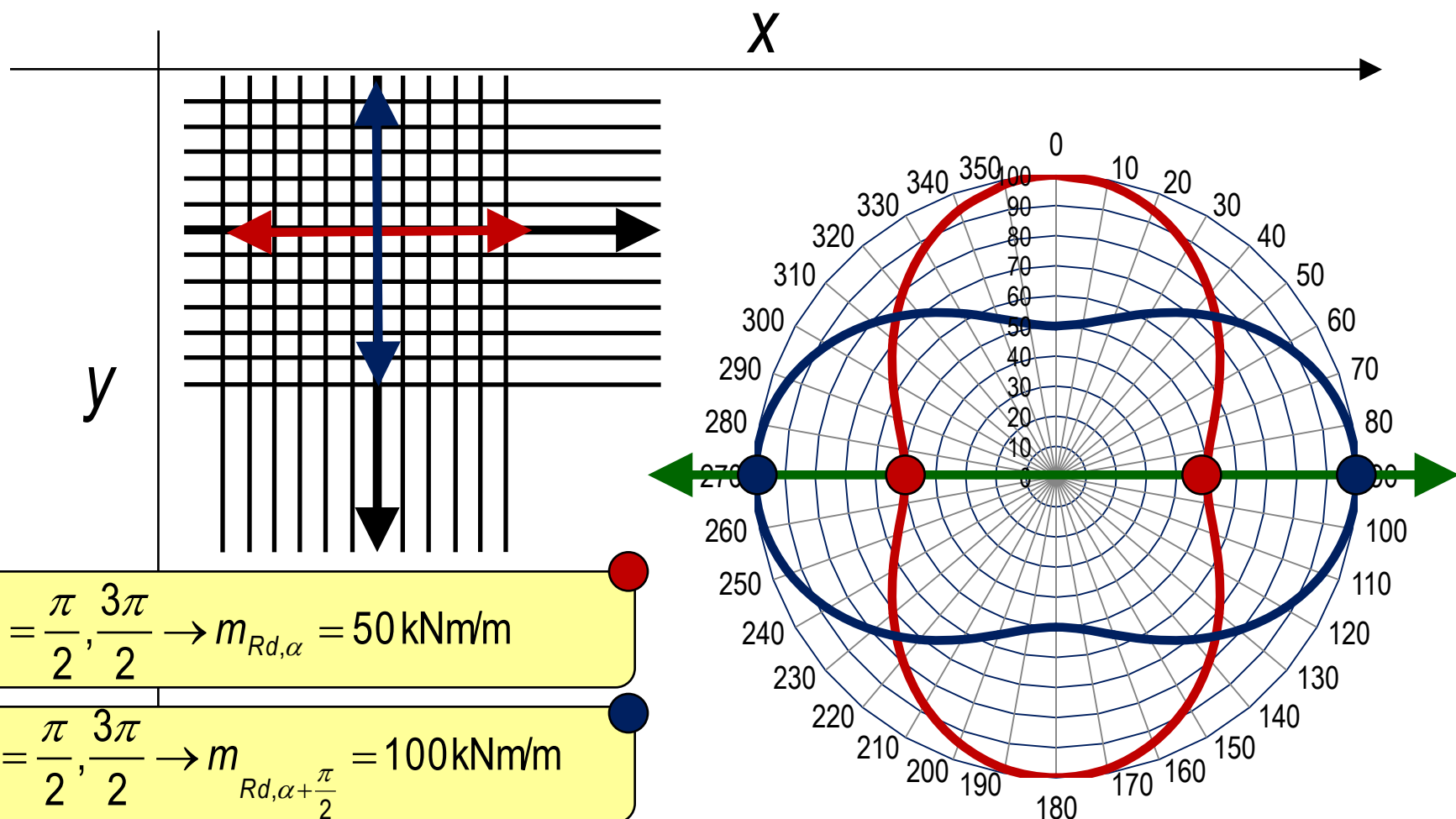
$$\alpha = \frac{\pi}{3}, \frac{4\pi}{3} \rightarrow m_{Rd,\alpha} = 62,50 \text{ kNm/m}$$

$$\alpha = \frac{\pi}{3}, \frac{4\pi}{3} \rightarrow m_{Rd,\alpha+\frac{\pi}{2}} = 87,50 \text{ kNm/m}$$



Ultimate moment of a two-way slab in α direction

Kétirányban vasalt lemez α irányú törőnyomatéka



Simplified solution for moments of two-way slabs

Kétirányban teherviselő lemezek közelítő megoldásai

A lemez differenciálegyenletének alkalmazása a gyakorlati tervezés során még a legegyszerűbb esetben is nehézkes.

Simplified solution for moments of two-way slabs

Kétirányban teherviselő lemezek közelítő megoldásai

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Közelítő megoldások alkalmazása terjedt el.

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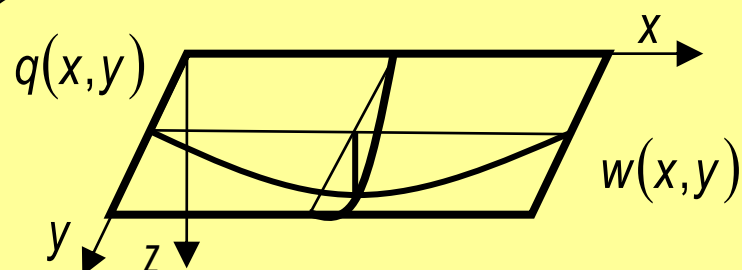


- **Klasszikus tartósáv módszer.**
- **Marcus-féle tartósáv módszer (Marcus táblázatok)**
- **Méretezési táblázatok alkalmazása**
- **Hatásfelületek**
- **Végeselem módszer - VEM**

Classical strip method

Klasszikus tartósáv módszer

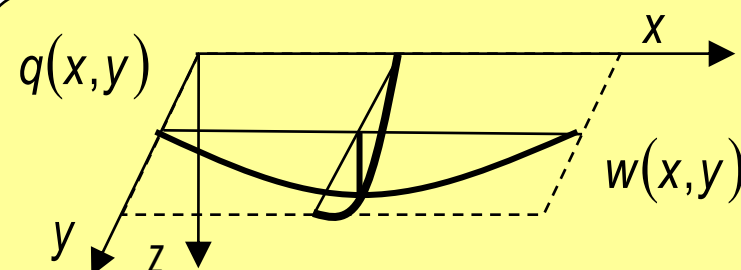
Kirchhoff-féle lemez



$$\frac{\partial^4 w}{\partial x^4} + 2 \cdot \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q(x,y)}{k}$$

$$\frac{\partial^2 m_x}{\partial x^2} + 2 \cdot \frac{\partial^2 m_{xy}}{\partial x \partial y} + \frac{\partial^2 m_y}{\partial y^2} = -q(x,y)$$

Csavarásmentes tartórács



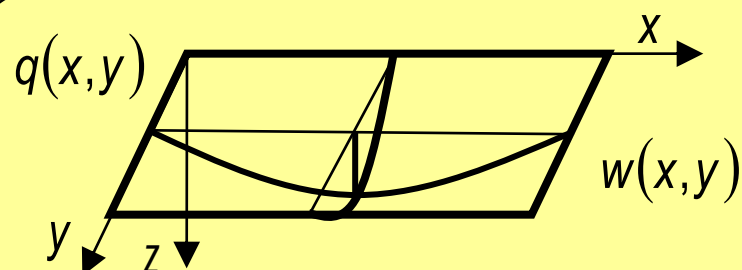
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Classical strip method

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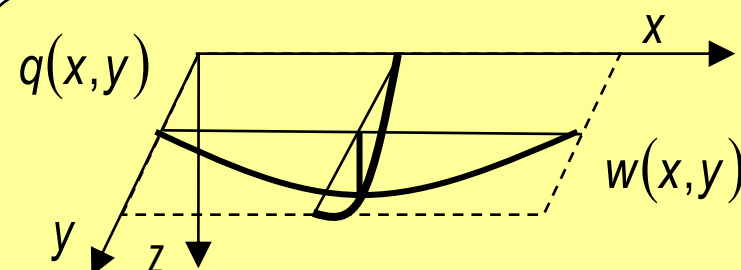
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Csavarásmentes tartórács



$$\frac{\partial^2 m_x}{\partial x^2} + \frac{\partial^2 m_y}{\partial y^2} = -q(x,y)$$

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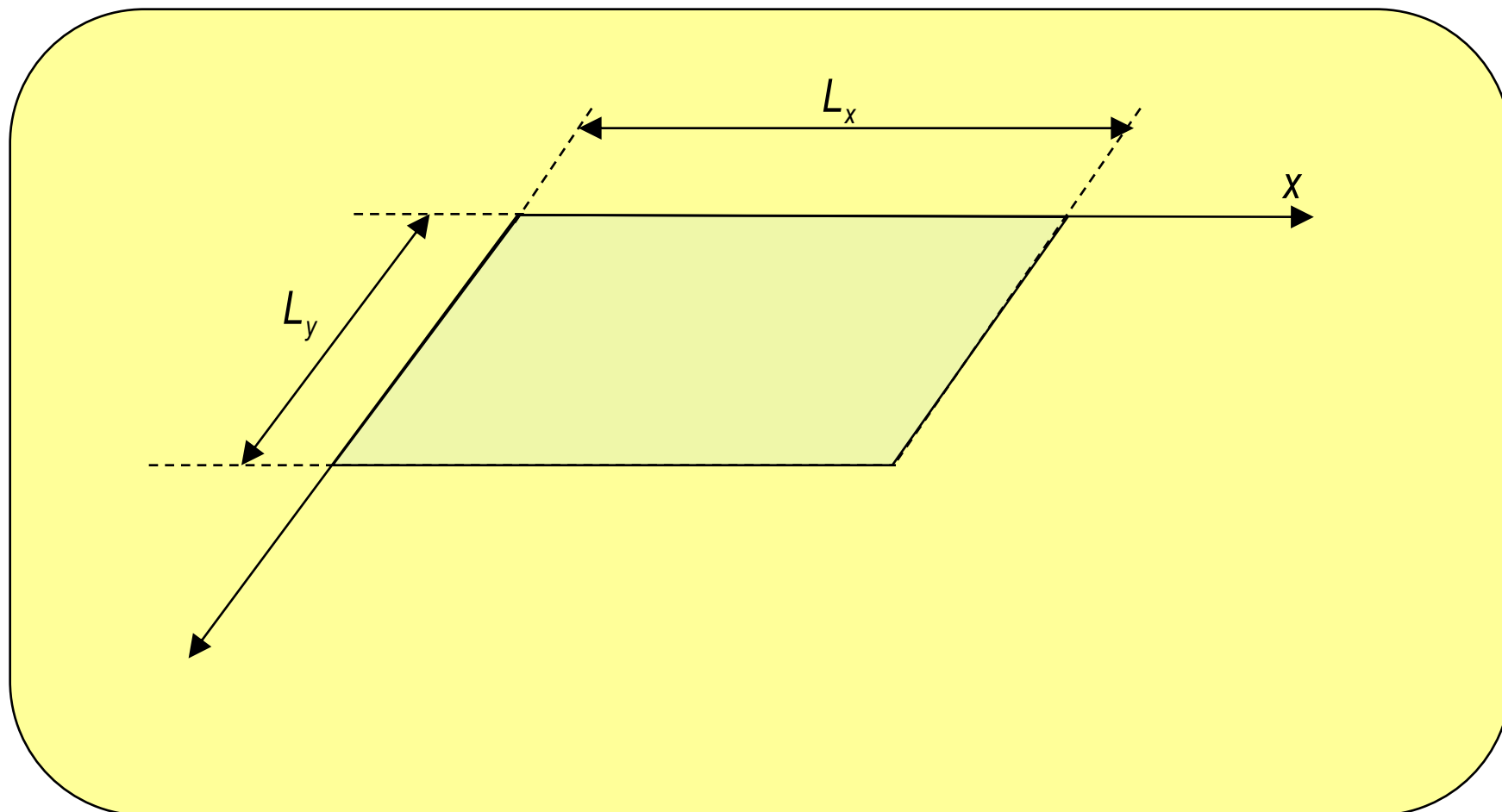
$$q(x,y) = q(x) + q(y)$$

$$\frac{\partial^2 m_x}{\partial x^2} = -q(x) \quad \frac{\partial^2 m_y}{\partial y^2} = -q(y)$$

Classical strip method

Klasszikus tartósáv módszer

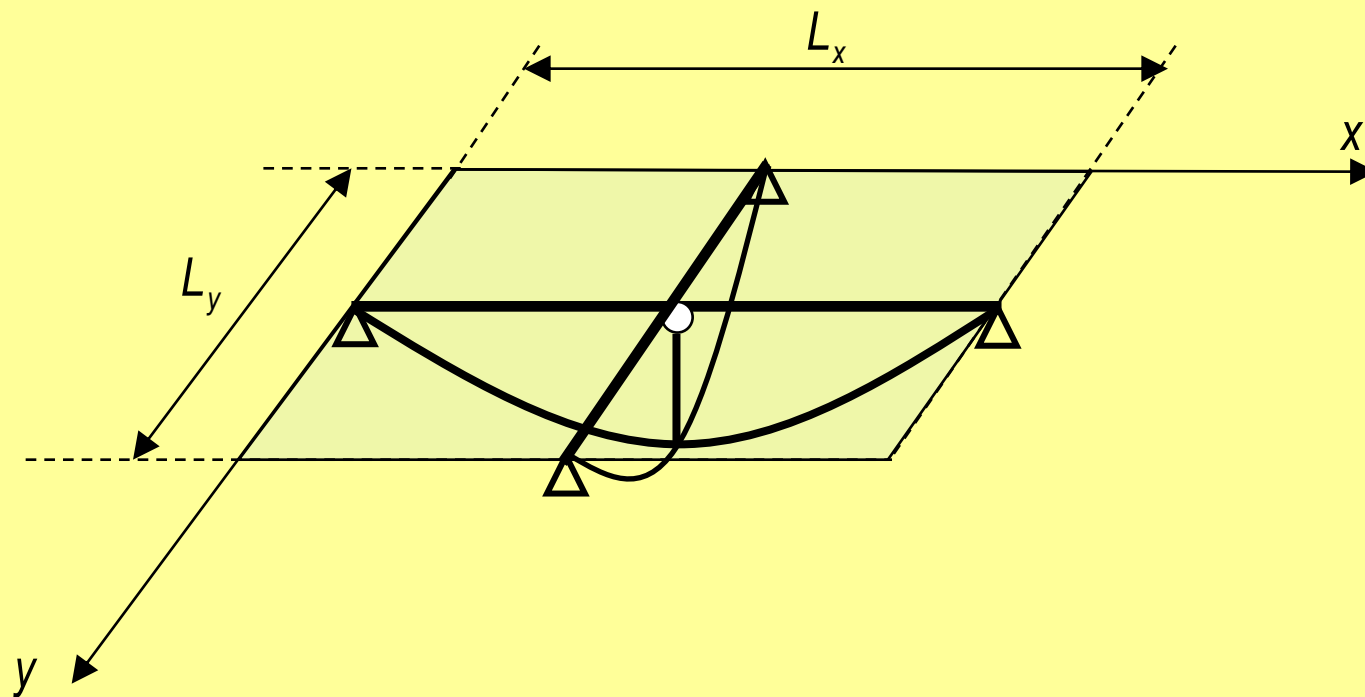
Csavarásmentes tartórács



Classical strip method

Klasszikus tartósáv módszer

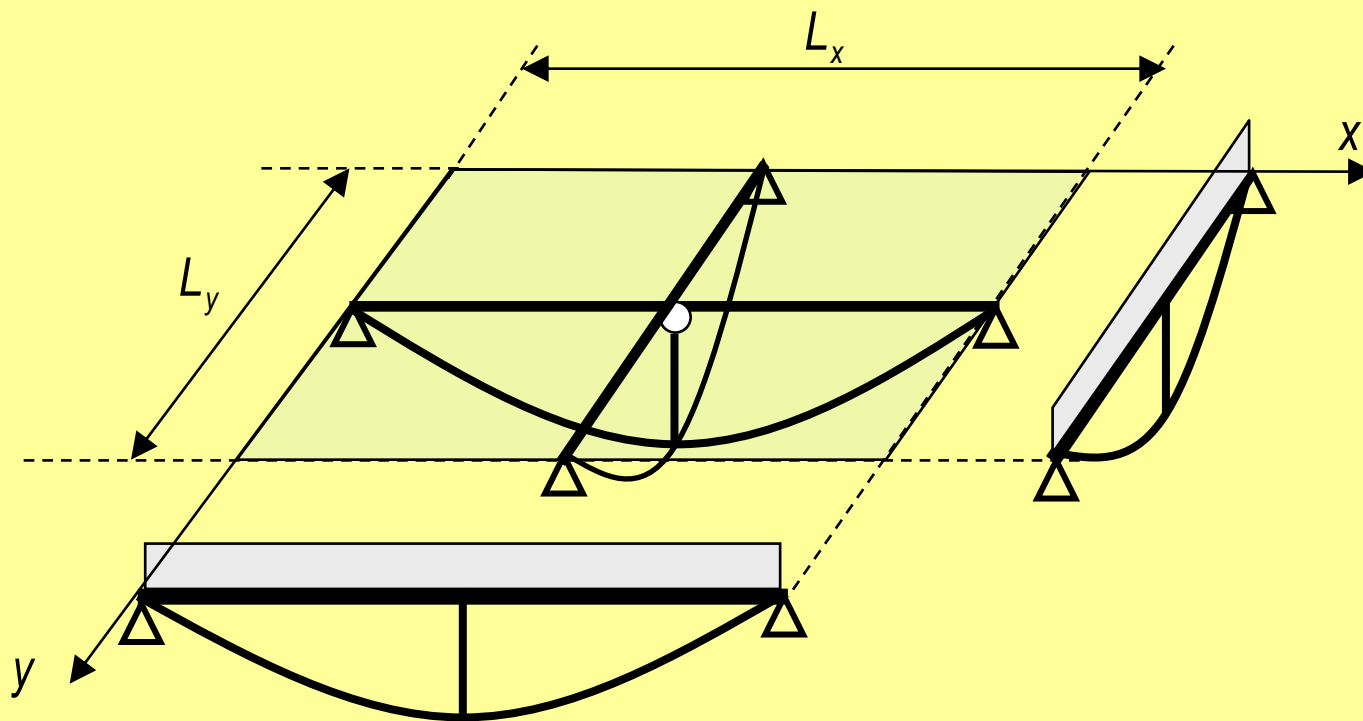
Csavarásmentes tartórács



Classical strip method

Klasszikus tartósáv módszer

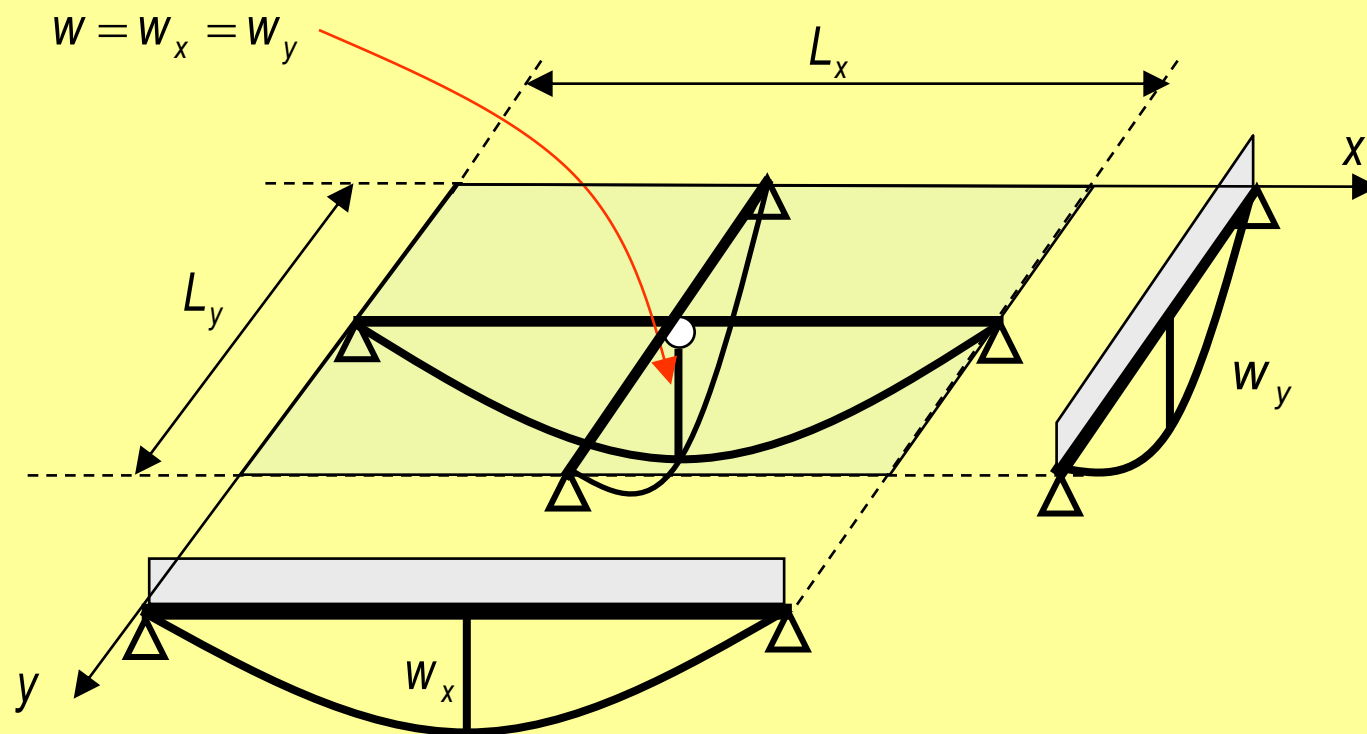
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Classical strip method

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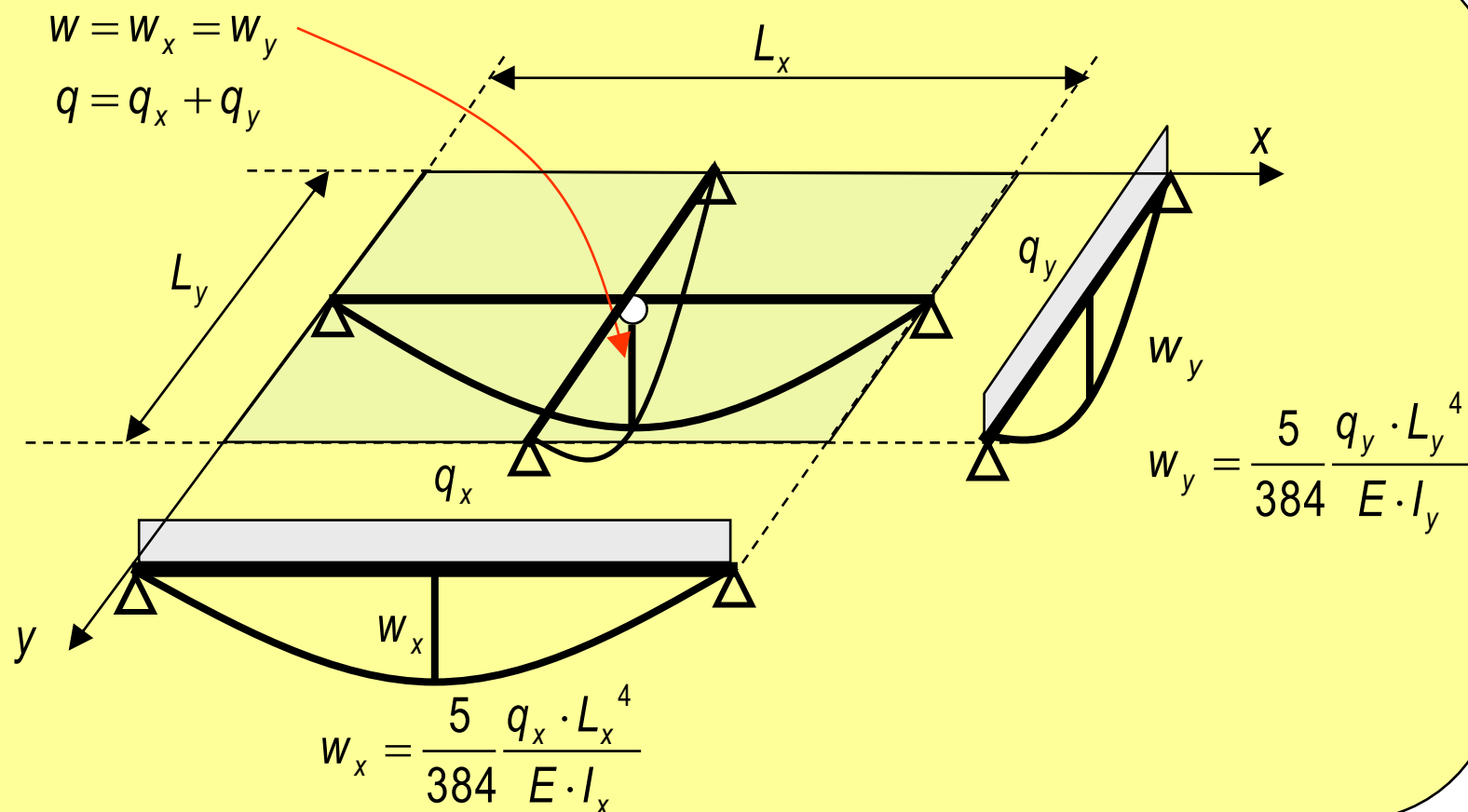
Csavarásmentes tartórács



Classical strip method

Klasszikus tartósáv módszer

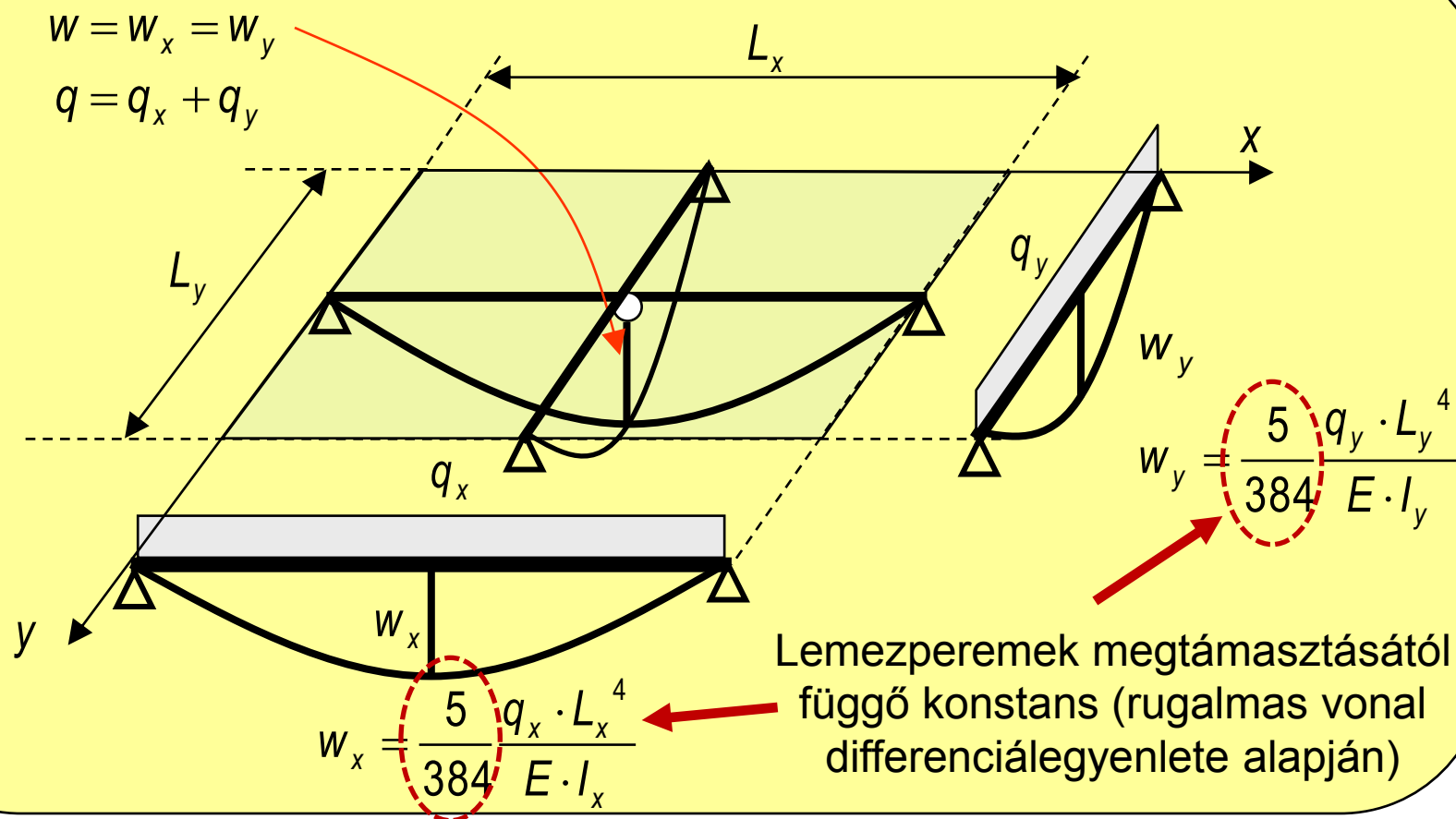
Csavarásmentes tartórács



Classical strip method

Klasszikus tartósáv módszer

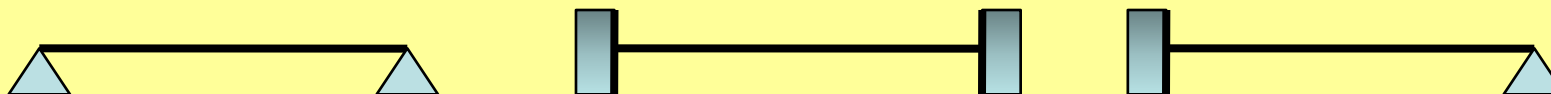
Csavarásmentes tartórács



Classical strip method

Klasszikus tartósáv módszer

Megtámasztási viszonyok



Lehajlás



$$w = \frac{5}{384} \frac{q \cdot L^4}{E \cdot I}$$



$$w = \frac{1}{384} \frac{q \cdot L^4}{E \cdot I}$$



$$w = \frac{2}{384} \frac{q \cdot L^4}{E \cdot I}$$

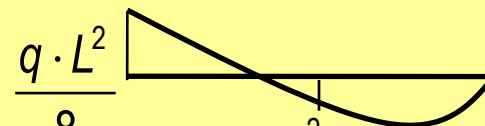
Hajlítónyomaték



$$\frac{q \cdot L^2}{8}$$



$$\frac{q \cdot L^2}{12} \quad \frac{q \cdot L^2}{24}$$

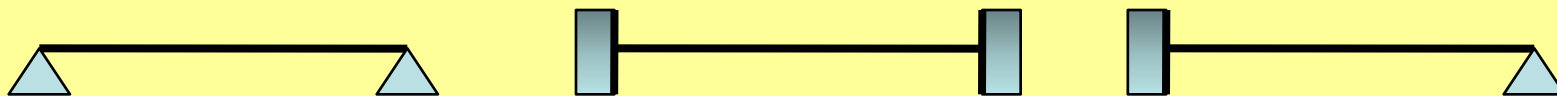


$$\frac{q \cdot L^2}{8} \quad \frac{q \cdot L^2}{16}$$

Classical strip method

Klasszikus tartósáv módszer

Megtámasztási viszonyok



Lehajlás

$$w = a_1 \cdot \frac{q \cdot L^4}{E \cdot I} = \frac{5}{384} \cdot \frac{q \cdot L^4}{E \cdot I}$$

$$w = a_2 \cdot \frac{q \cdot L^4}{E \cdot I} = \frac{1}{384} \cdot \frac{q \cdot L^4}{E \cdot I}$$

$$w = a_3 \cdot \frac{q \cdot L^4}{E \cdot I} = \frac{2}{384} \cdot \frac{q \cdot L^4}{E \cdot I}$$

Hajlítónyomaték

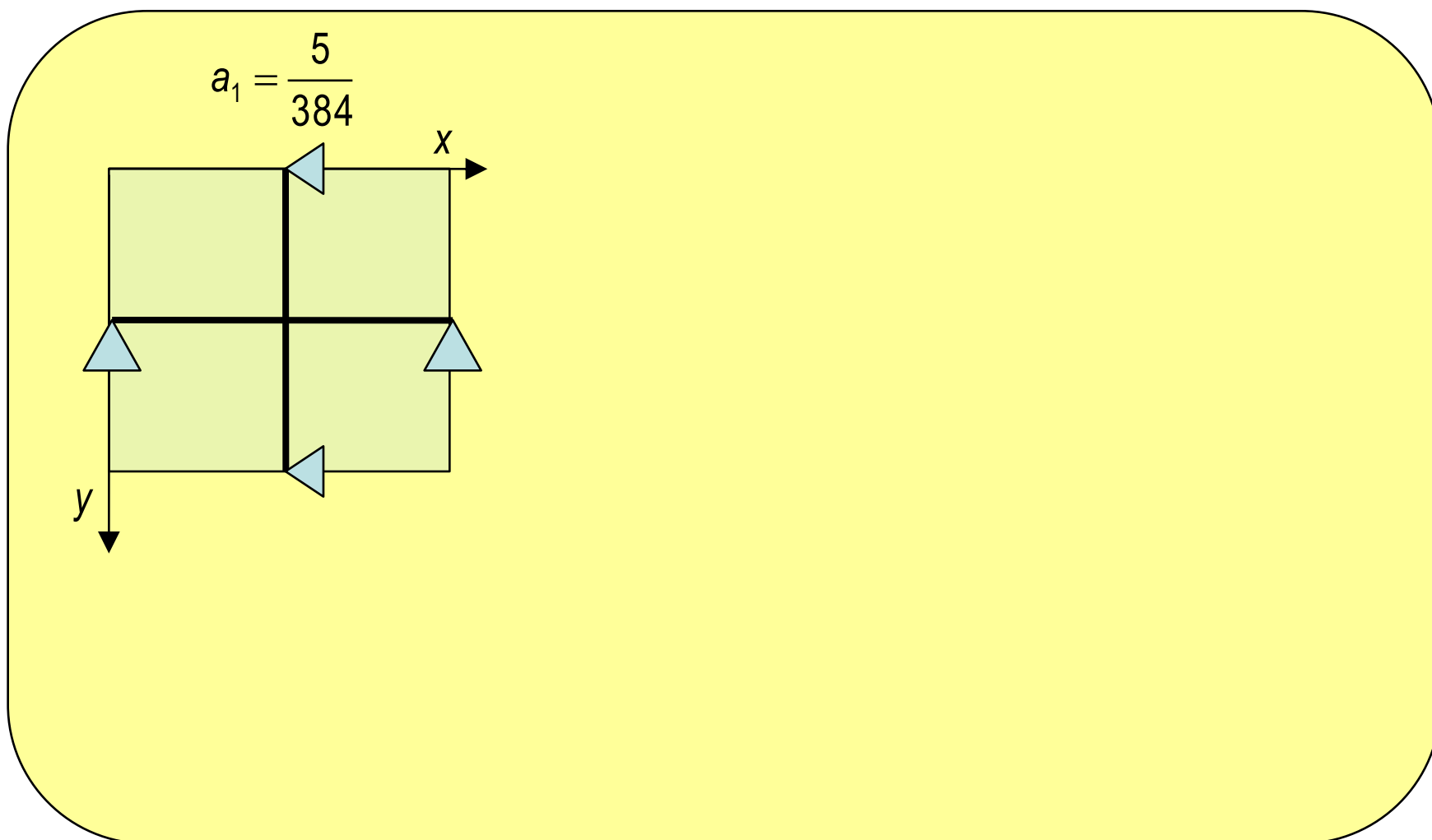
$$\frac{q \cdot L^2}{8}$$

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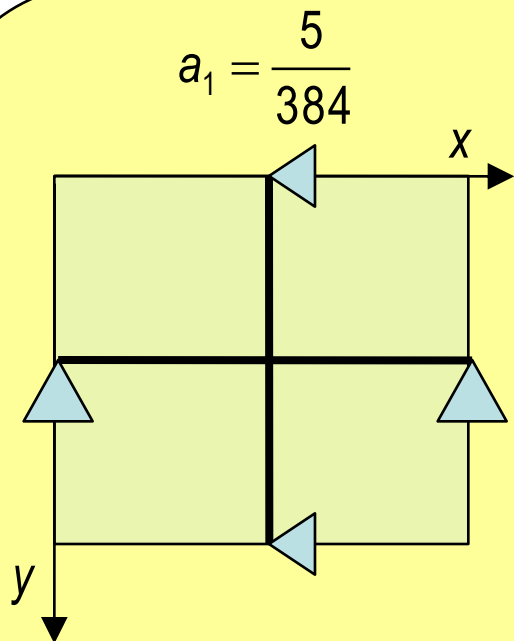
1. case: Slab simply supported on all four edges

1. eset: Peremlein csuklósan megtámasztott lemez



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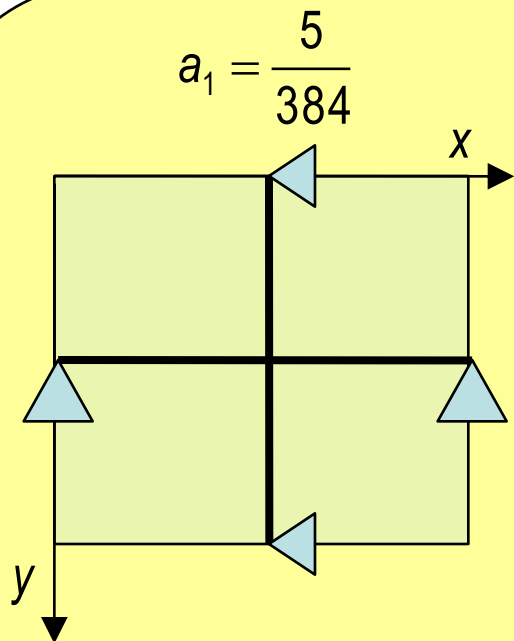
1. eset: Peremlein csuklósan megtámasztott lemez



$$w_x = a_1 \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = a_1 \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

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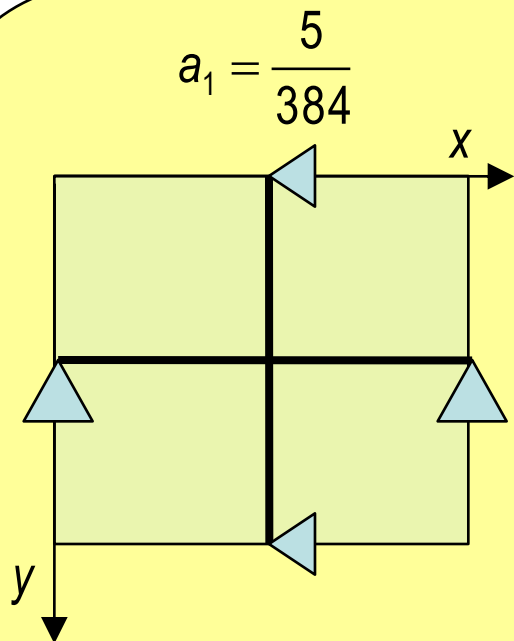


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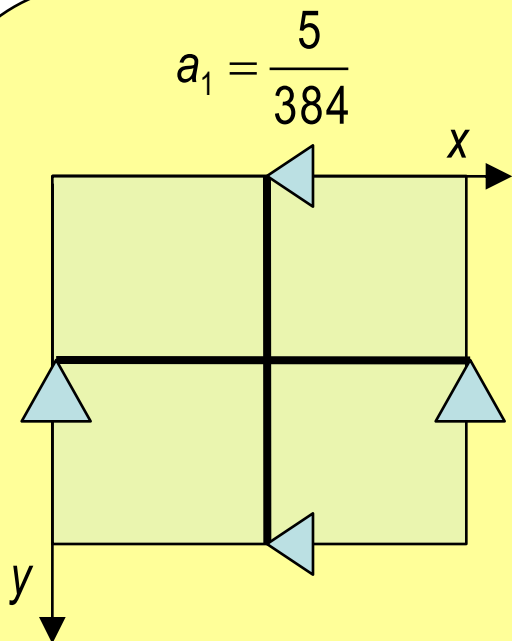
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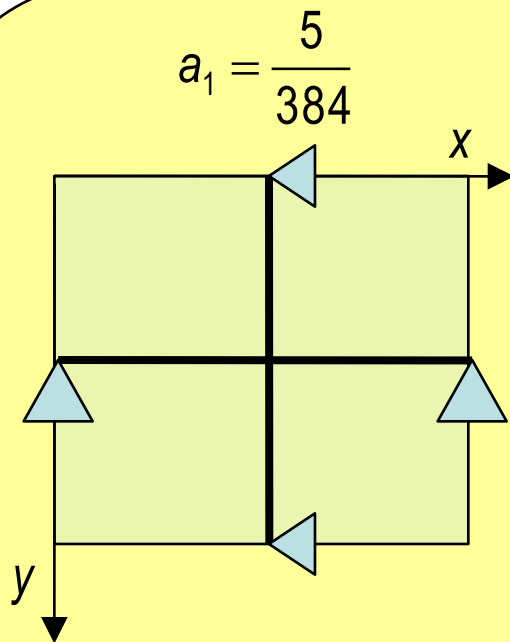
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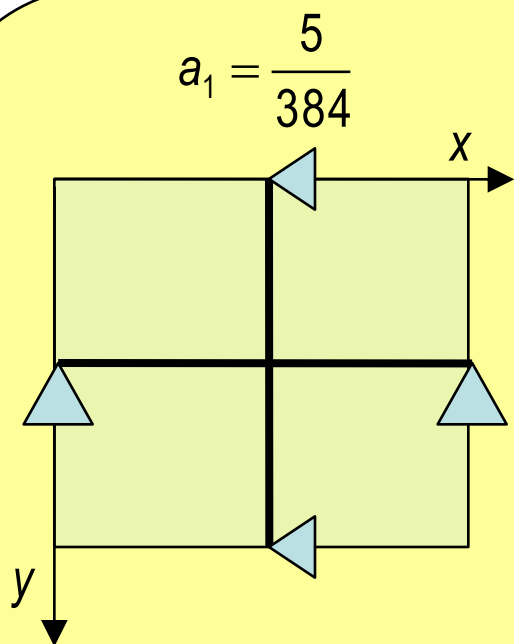
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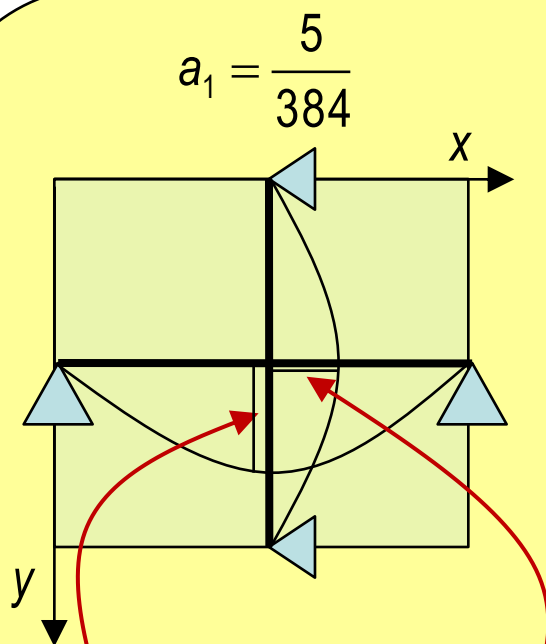
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$$q_x = \frac{\lambda^4}{1 + \lambda^4} \cdot q \quad q_y = \frac{1}{1 + \lambda^4} \cdot q$$

1. case: Slab simply supported on all four edges

1. eset: Peremlein csuklósan megtámasztott lemez



$$+m_{x,y} = \frac{q_{x,y} \cdot L_{x,y}^2}{8}$$

$$w_x = a_1 \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = a_1 \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

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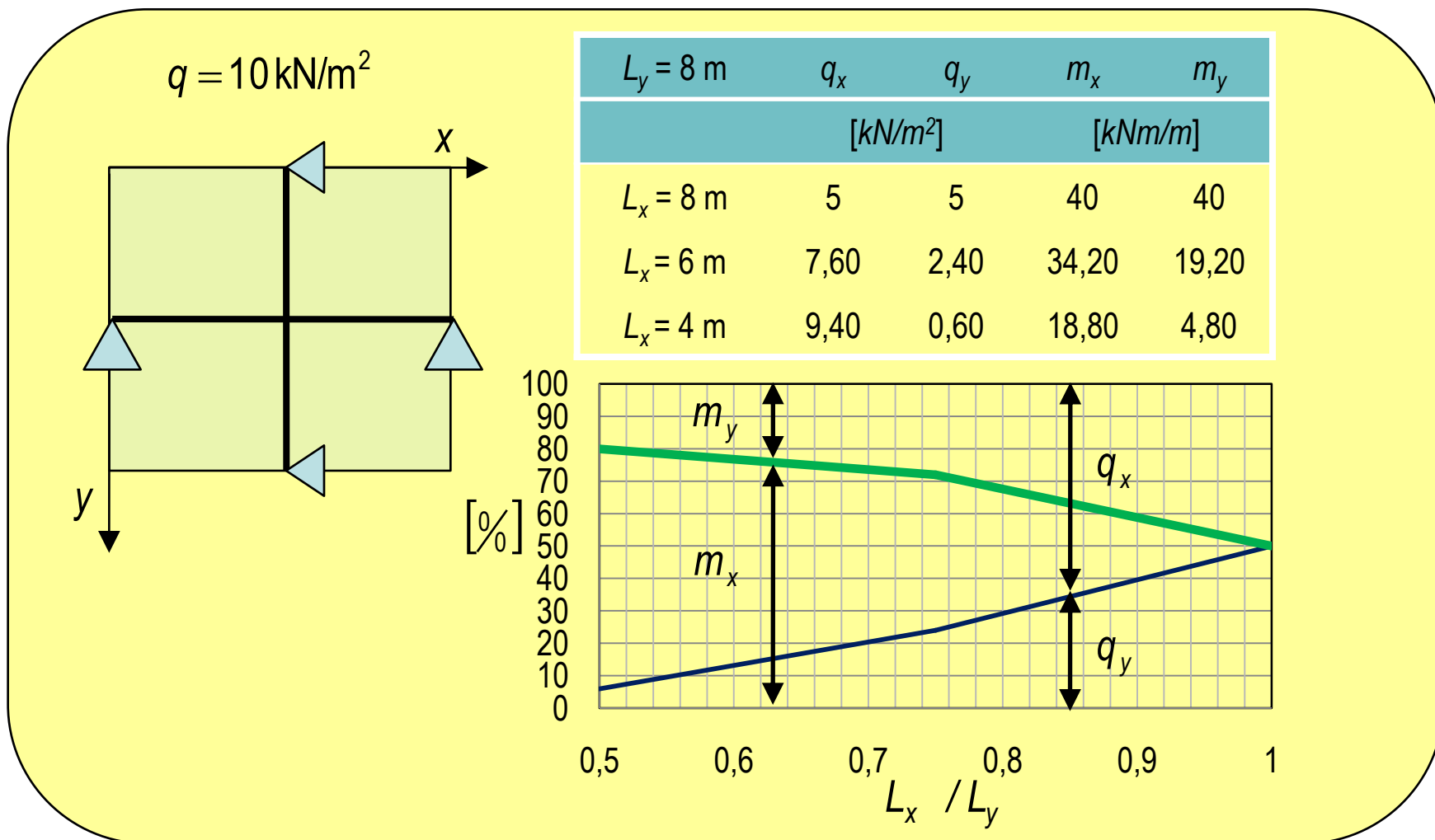
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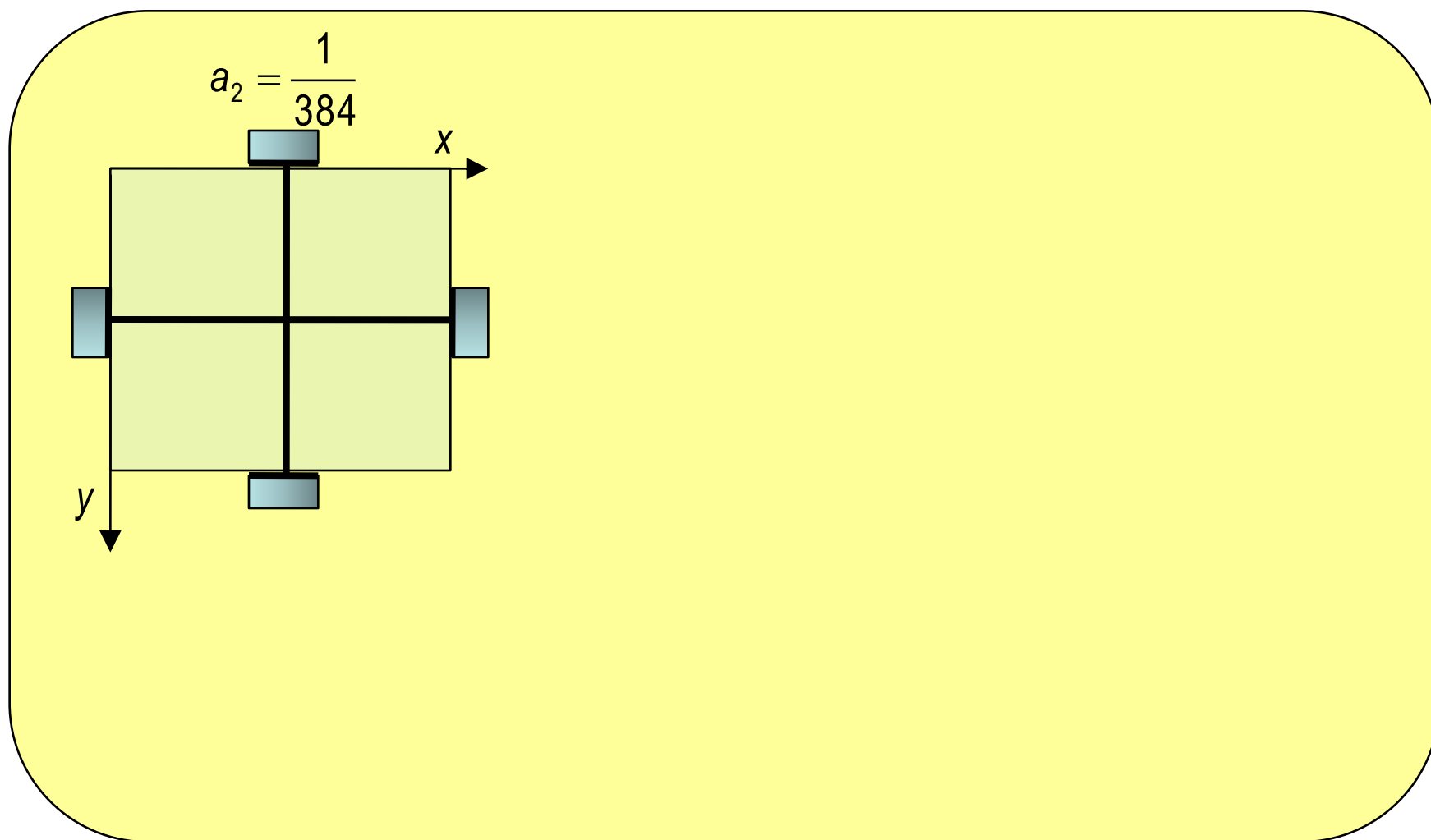
1. case: Slab simply supported on all four edges

1. eset: Peremlein csuklósan megtámasztott lemez



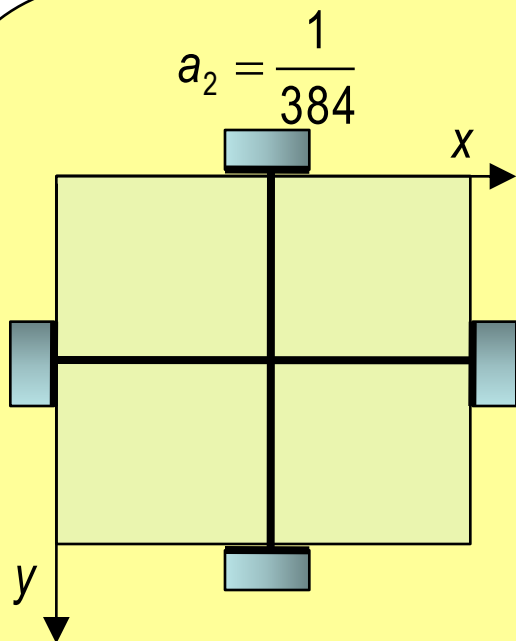
2. case: Slab supported on all four fix edges

2. eset: Peremlein befogott lemez



2. case: Slab supported on all four fix edges

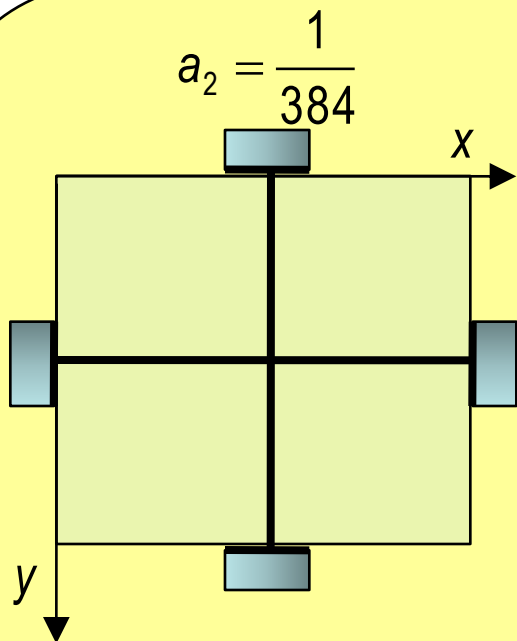
2. eset: Peremlein befogott lemez



$$w_x = a_2 \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = a_2 \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

2. case: Slab supported on all four fix edges

2. eset: Peremlein befogott lemez

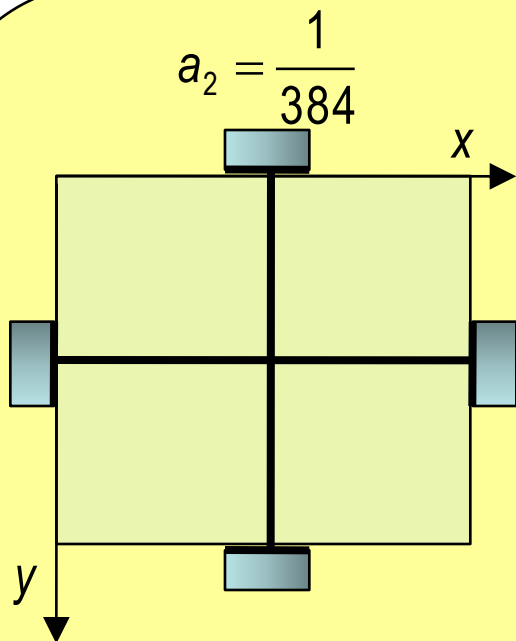


$$w_x = a_2 \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = a_2 \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

$$a_2 \cdot q_x \cdot L_x^4 = a_2 \cdot q_y \cdot L_y^4$$

2. case: Slab supported on all four fix edges

2. eset: Peremlein befogott lemez



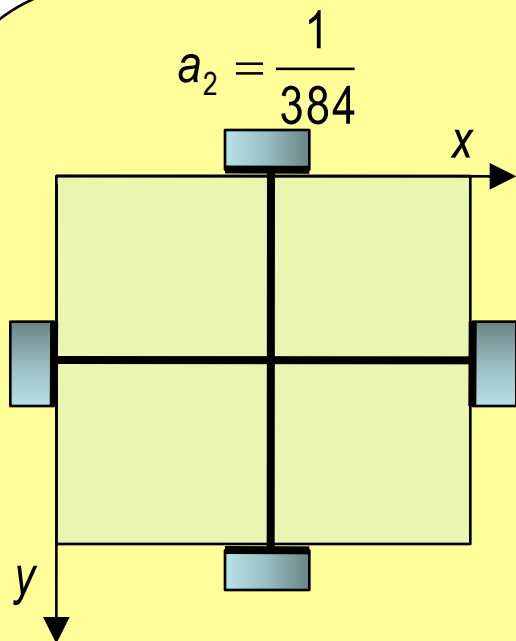
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2. eset: Peremlein befogott lemez



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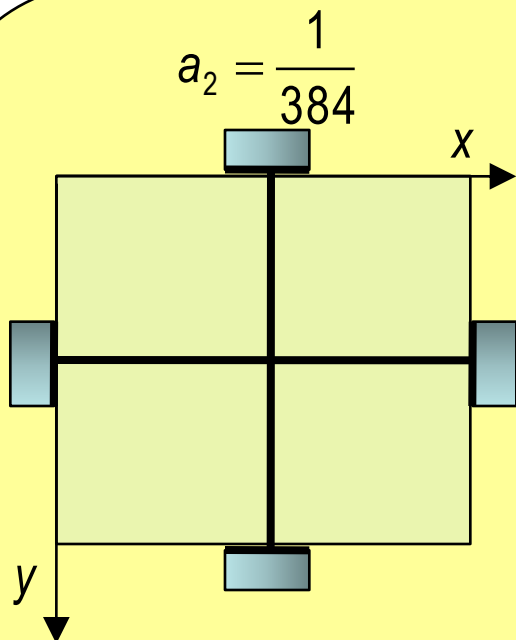
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2. case: Slab supported on all four fix edges

2. eset: Peremlein befogott lemez



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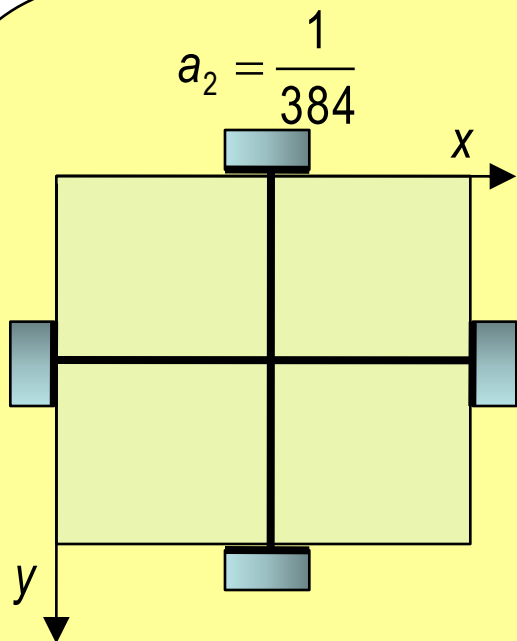
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2. eset: Peremlein befogott lemez



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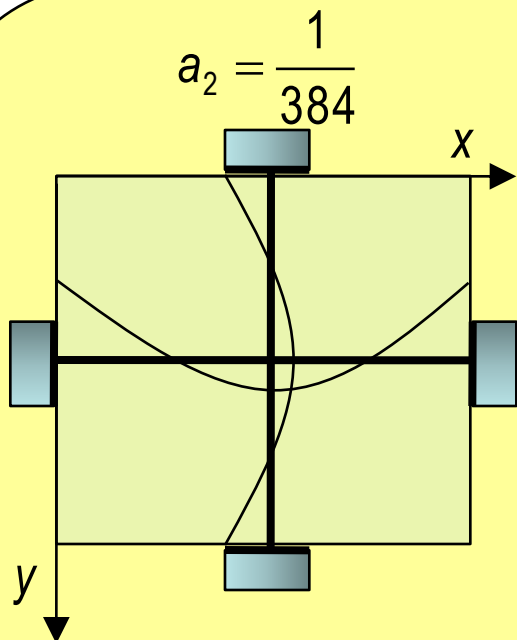
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2. case: Slab supported on all four fix edges

2. eset: Peremlein befogott lemez



$$+m_{x,y} = \frac{q_{x,y} \cdot L_{x,y}^2}{24}$$

$$-m_{x,y} = \frac{q_{x,y} \cdot L_{x,y}^2}{12}$$

$$w_x = a_2 \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = a_2 \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

$$a_2 \cdot q_x \cdot L_x^4 = a_2 \cdot q_y \cdot L_y^4$$

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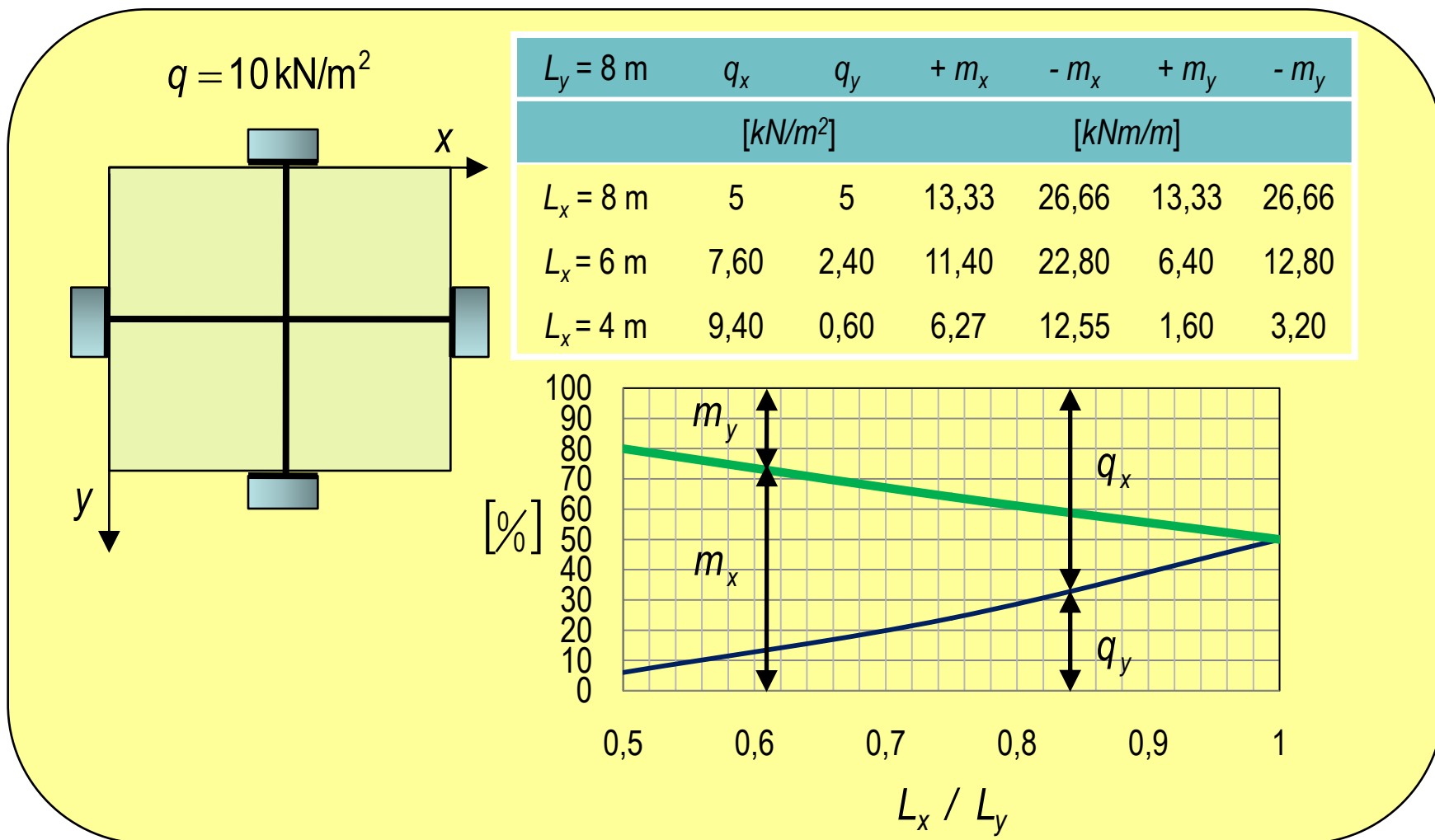
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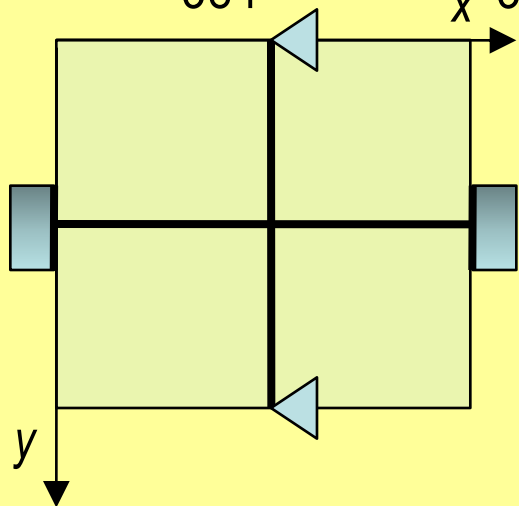
2. eset: Peremlein befogott lemez



3. case: Slab with two fix and two simple edges

3. eset: Szemben lévő peremlein befogott ill. csuklós lemez

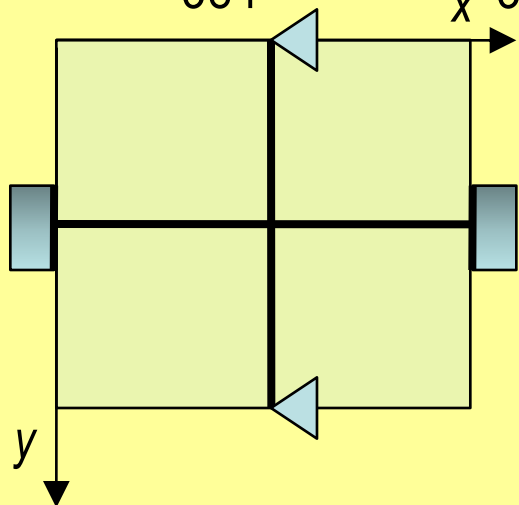
$$a_x = a_2 = \frac{1}{384} \quad a_y = a_1 = \frac{5}{384}$$



3. case: Slab with two fix and two simple edges

3. eset: Szemben lévő peremlein befogott ill. csuklós lemez

$$a_x = a_2 = \frac{1}{384} \quad a_y = a_1 = \frac{5}{384}$$

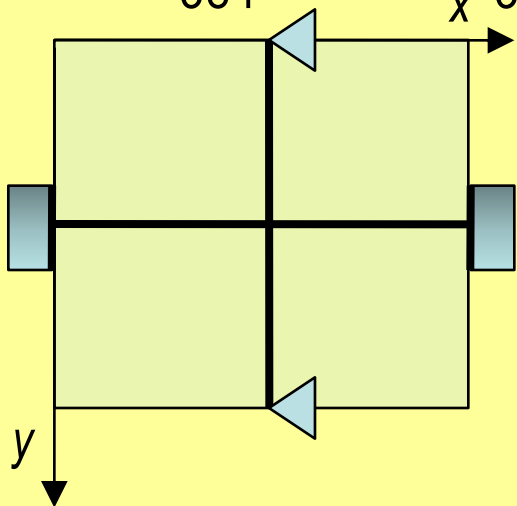


$$w_x = \frac{1}{384} \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = \frac{5}{384} \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

3. case: Slab with two fix and two simple edges

3. eset: Szemben lévő peremlein befogott ill. csuklós lemez

$$a_x = a_2 = \frac{1}{384} \quad a_y = a_1 = \frac{5}{384}$$



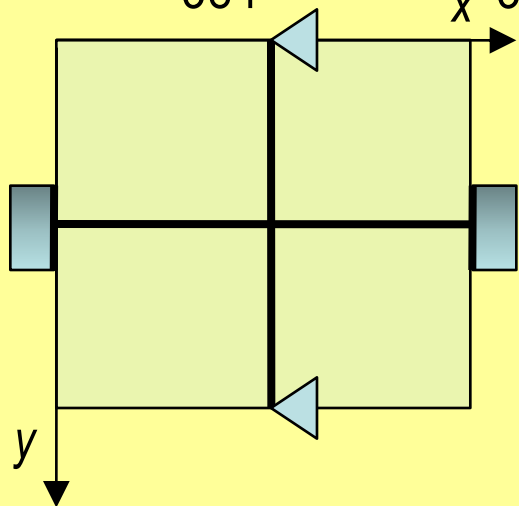
$$w_x = \frac{1}{384} \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = \frac{5}{384} \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

$$1 \cdot q_x \cdot L_x^4 = 5 \cdot q_y \cdot L_y^4$$

3. case: Slab with two fix and two simple edges

3. eset: Szemben lévő peremlein befogott ill. csuklós lemez

$$a_x = a_2 = \frac{1}{384} \quad a_y = a_1 = \frac{5}{384}$$



$$w_x = \frac{1}{384} \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = \frac{5}{384} \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

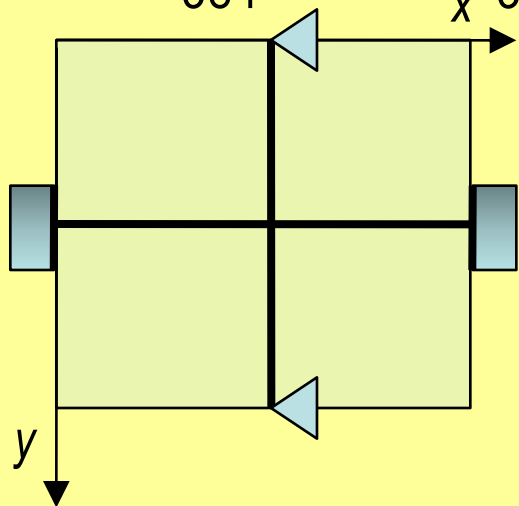
$$1 \cdot q_x \cdot L_x^4 = 5 \cdot q_y \cdot L_y^4$$

$$\frac{a_2}{a_1} \cdot \frac{q_x}{q_y} = \frac{1}{5} \cdot \frac{q_x}{q_y} = \left(\frac{L_y}{L_x} \right)^4 = \lambda^4$$

3. case: Slab with two fix and two simple edges

3. eset: Szemben lévő peremlein befogott ill. csuklós lemez

$$a_x = a_2 = \frac{1}{384} \quad a_y = a_1 = \frac{5}{384}$$



$$w_x = \frac{1}{384} \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = \frac{5}{384} \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

$$1 \cdot q_x \cdot L_x^4 = 5 \cdot q_y \cdot L_y^4$$

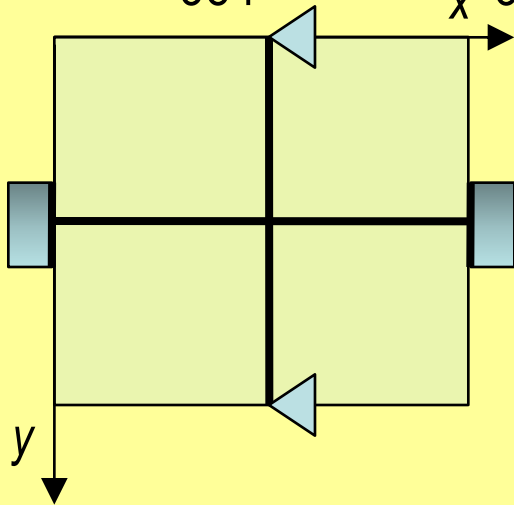
$$\frac{a_2}{a_1} \cdot \frac{q_x}{q_y} = \frac{1}{5} \cdot \frac{q_x}{q_y} = \left(\frac{L_y}{L_x} \right)^4 = \lambda^4$$

$$q_x = \frac{a_1}{a_2} \cdot \lambda^4 \cdot q_y \quad q_y = \frac{a_2}{a_1} \cdot \frac{1}{\lambda^4} \cdot q_x$$

3. case: Slab with two fix and two simple edges

3. eset: Szemben lévő peremlein befogott ill. csuklós lemez

$$a_x = a_2 = \frac{1}{384} \quad a_y = a_1 = \frac{5}{384}$$



$$w_x = \frac{1}{384} \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = \frac{5}{384} \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

$$1 \cdot q_x \cdot L_x^4 = 5 \cdot q_y \cdot L_y^4$$

$$\frac{a_2}{a_1} \cdot \frac{q_x}{q_y} = \frac{1}{5} \cdot \frac{q_x}{q_y} = \left(\frac{L_y}{L_x} \right)^4 = \lambda^4$$

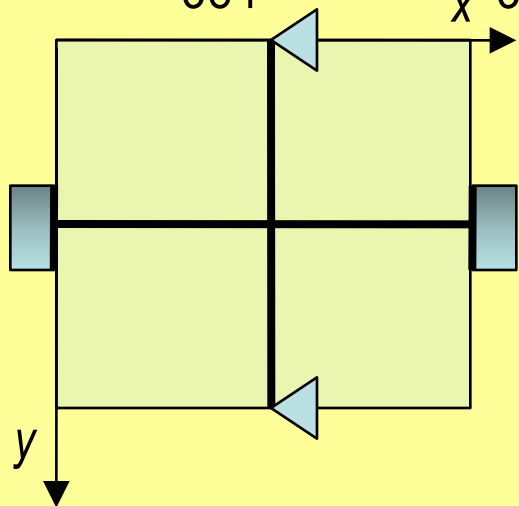
$$q_x = \frac{a_1}{a_2} \cdot \lambda^4 \cdot q_y \quad q_y = \frac{a_2}{a_1} \cdot \frac{1}{\lambda^4} \cdot q_x$$

$$q = q_x + q_y$$

3. case: Slab with two fix and two simple edges

3. eset: Szemben lévő peremlein befogott ill. csuklós lemez

$$a_x = a_2 = \frac{1}{384} \quad a_y = a_1 = \frac{5}{384}$$



$$w_x = \frac{1}{384} \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = \frac{5}{384} \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

$$1 \cdot q_x \cdot L_x^4 = 5 \cdot q_y \cdot L_y^4$$

$$\frac{a_2}{a_1} \cdot \frac{q_x}{q_y} = \frac{1}{5} \cdot \frac{q_x}{q_y} = \left(\frac{L_y}{L_x} \right)^4 = \lambda^4$$

$$q_x = \frac{a_1}{a_2} \cdot \lambda^4 \cdot q_y \quad q_y = \frac{a_2}{a_1} \cdot \frac{1}{\lambda^4} \cdot q_x$$

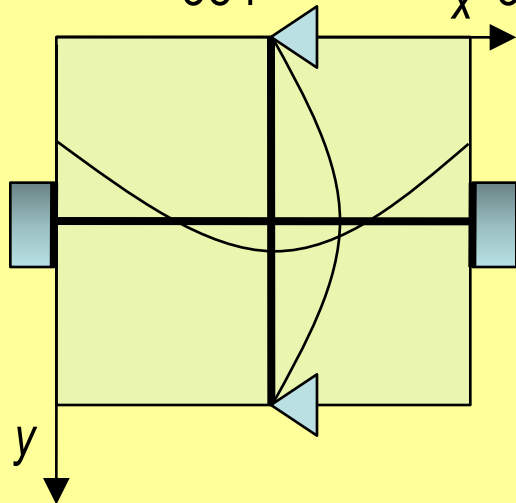
$$q = q_x + q_y$$

$$q_x = \frac{\lambda^4}{1/5 + \lambda^4} \cdot q \quad q_y = \frac{1/5}{1/5 + \lambda^4} \cdot q$$

3. case: Slab with two fix and two simple edges

3. eset: Szemben lévő peremlein befogott ill. csuklós lemez

$$a_x = a_2 = \frac{1}{384} \quad a_y = a_1 = \frac{5}{384}$$



$$+m_x = \frac{q_x \cdot L_x^2}{24} \quad -m_x = -2 \cdot m_x$$

$$+m_y = \frac{q_y \cdot L_y^2}{8}$$

$$w_x = \frac{1}{384} \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = \frac{5}{384} \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

$$1 \cdot q_x \cdot L_x^4 = 5 \cdot q_y \cdot L_y^4$$

$$\frac{a_2}{a_1} \cdot \frac{q_x}{q_y} = \frac{1}{5} \cdot \frac{q_x}{q_y} = \left(\frac{L_y}{L_x} \right)^4 = \lambda^4$$

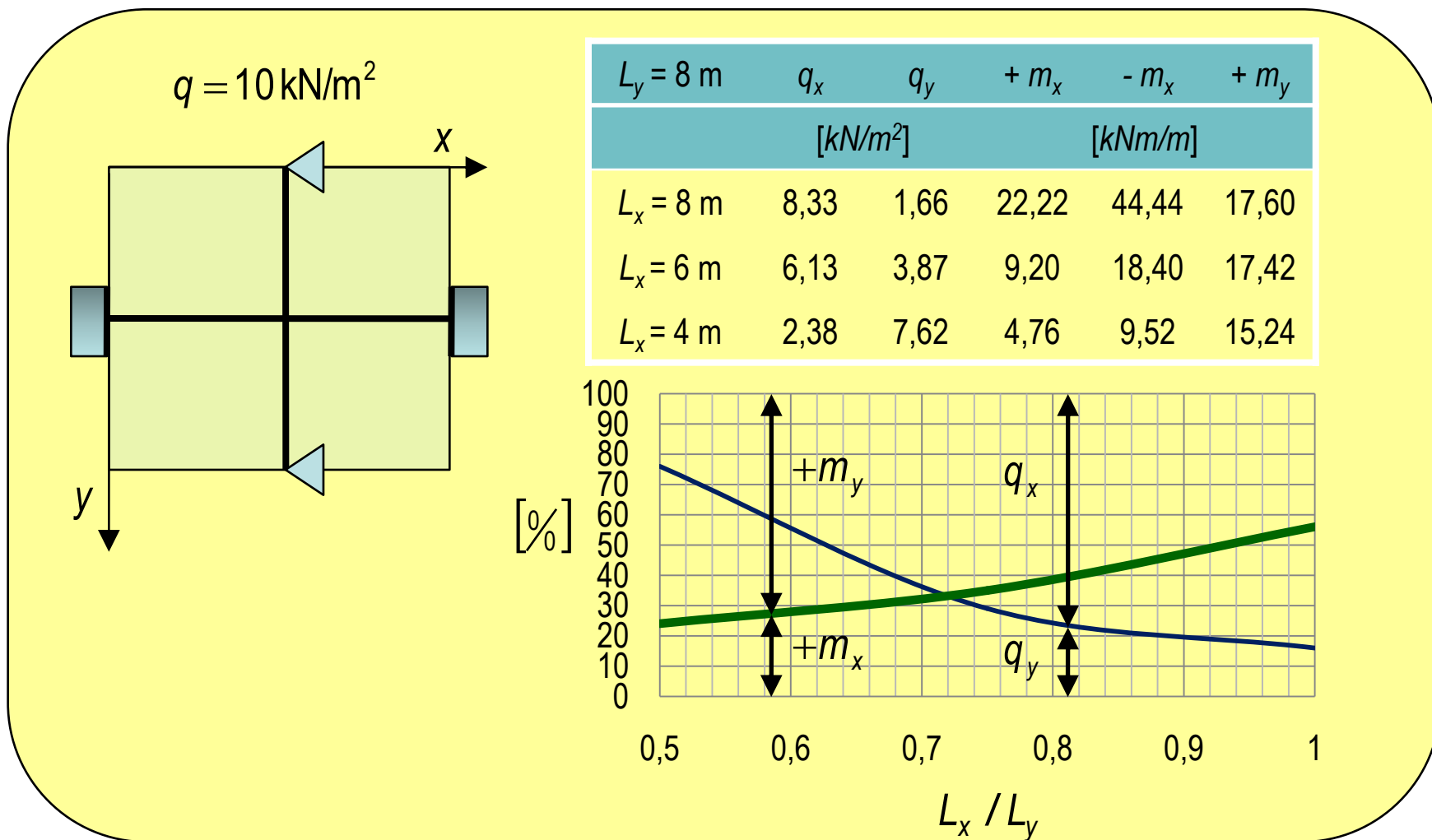
$$q_x = \frac{a_1}{a_2} \cdot \lambda^4 \cdot q_y \quad q_y = \frac{a_2}{a_1} \cdot \frac{1}{\lambda^4} \cdot q_x$$

$$q = q_x + q_y$$

$$q_x = \frac{\lambda^4}{1/5 + \lambda^4} \cdot q \quad q_y = \frac{1/5}{1/5 + \lambda^4} \cdot q$$

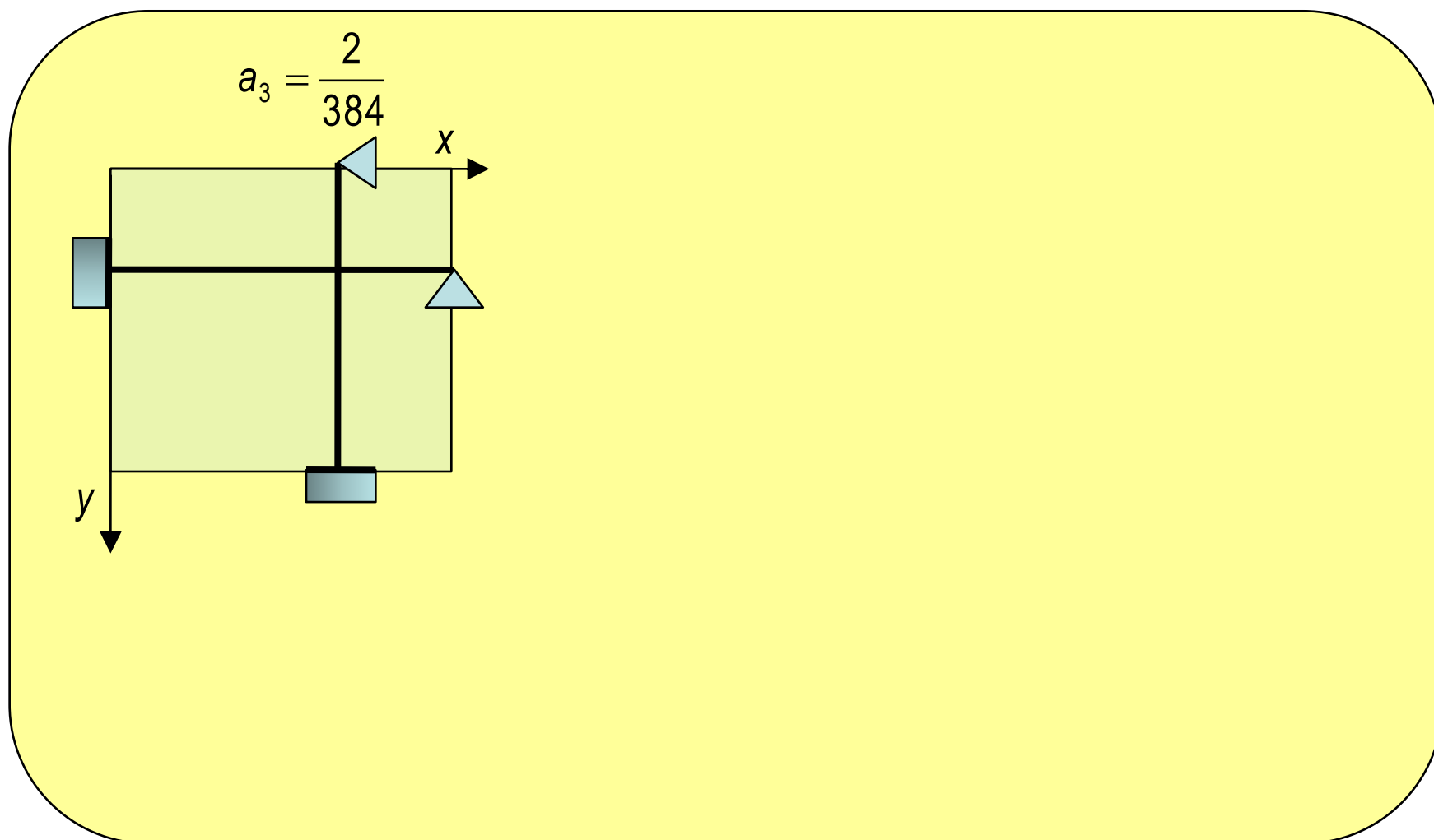
3. case: Slab with two fix and two simple edges

3. eset: Szemben lévő peremlein befogott ill. csuklós lemez



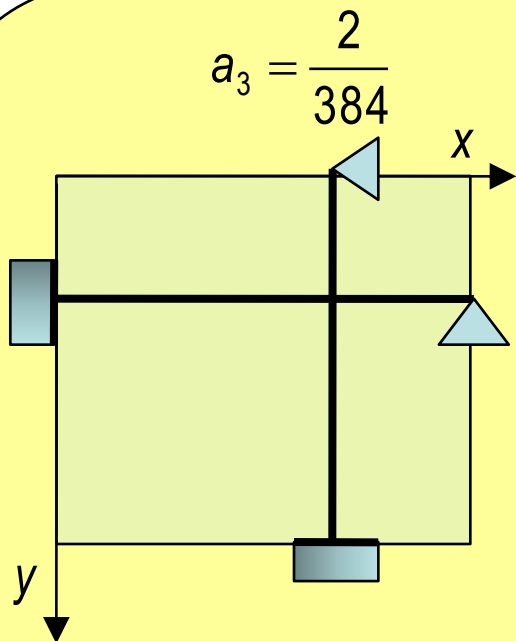
4. case: Slab with two fix and two simple edges

4. eset: Szemben lévő peremein befogott-csuklós lemez



4. case: Slab with two fix and two simple edges

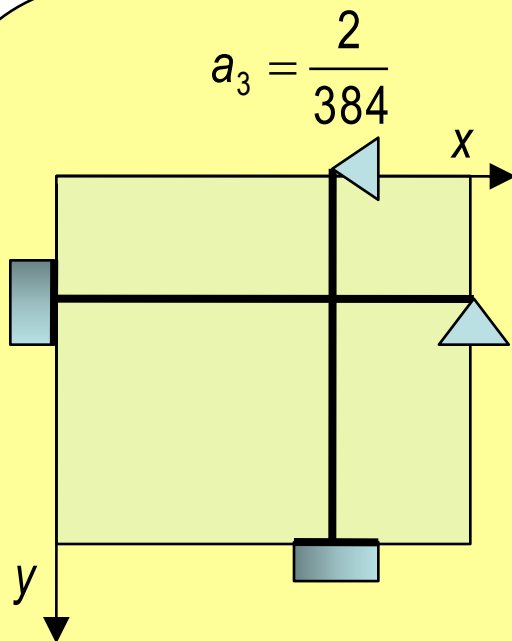
4. eset: Szemben lévő peremlein befogott-csuklós lemez



$$w_x = \frac{2}{384} \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = \frac{2}{384} \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

4. case: Slab with two fix and two simple edges

4. eset: Szemben lévő peremlein befogott-csuklós lemez

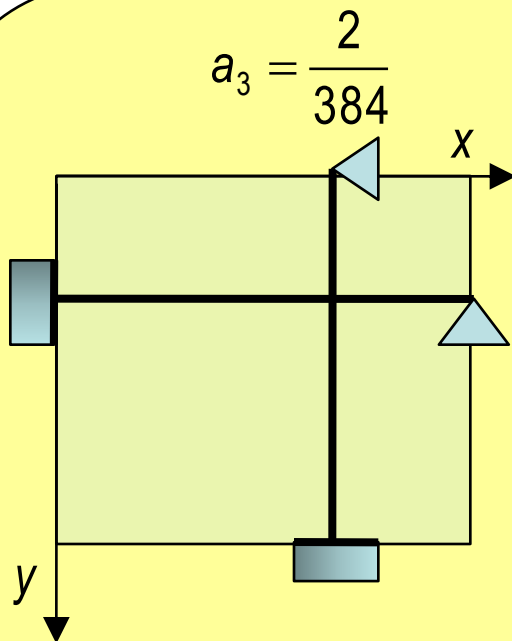


$$w_x = \frac{2}{384} \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = \frac{2}{384} \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

$$q_x \cdot L_x^4 = q_y \cdot L_y^4$$

4. case: Slab with two fix and two simple edges

4. eset: Szemben lévő peremlein befogott-csuklós lemez



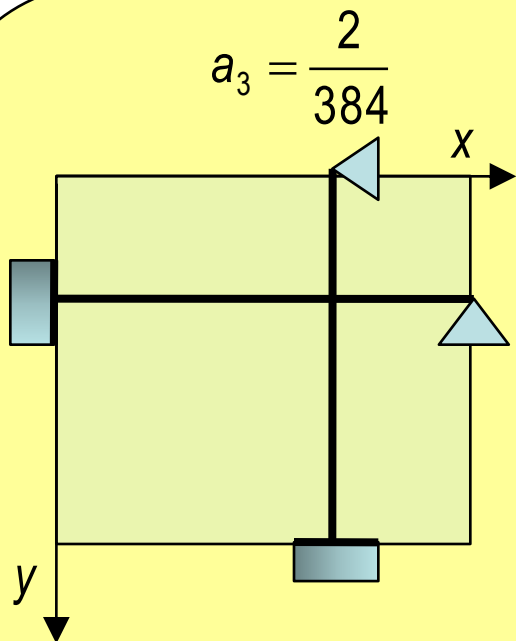
$$w_x = \frac{2}{384} \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = \frac{2}{384} \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

$$q_x \cdot L_x^4 = q_y \cdot L_y^4$$

$$\frac{a_3}{a_3} \cdot \frac{q_x}{q_y} = \frac{q_x}{q_y} = \left(\frac{L_y}{L_x} \right)^4 = \lambda^4$$

4. case: Slab with two fix and two simple edges

4. eset: Szemben lévő peremein befogott-csuklós lemez



$$w_x = \frac{2}{384} \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = \frac{2}{384} \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

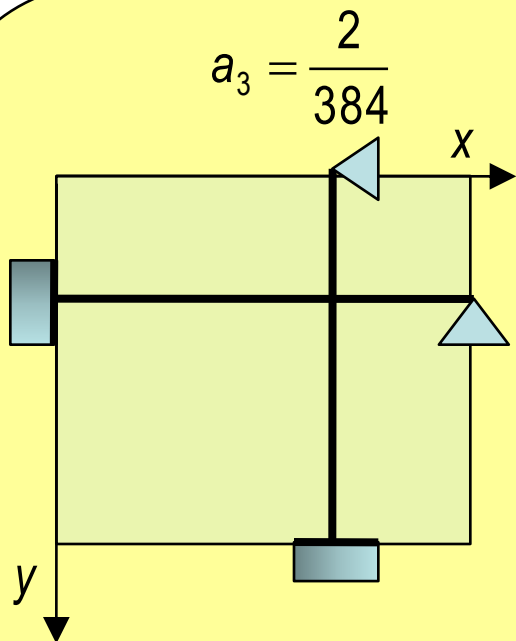
$$q_x \cdot L_x^4 = q_y \cdot L_y^4$$

$$\frac{a_3}{a_3} \cdot \frac{q_x}{q_y} = \frac{q_x}{q_y} = \left(\frac{L_y}{L_x} \right)^4 = \lambda^4$$

$$q_x = \lambda^4 \cdot q_y \qquad q_y = \frac{1}{\lambda^4} \cdot q_x$$

4. case: Slab with two fix and two simple edges

4. eset: Szemben lévő peremlein befogott-csuklós lemez



$$w_x = \frac{2}{384} \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = \frac{2}{384} \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

$$q_x \cdot L_x^4 = q_y \cdot L_y^4$$

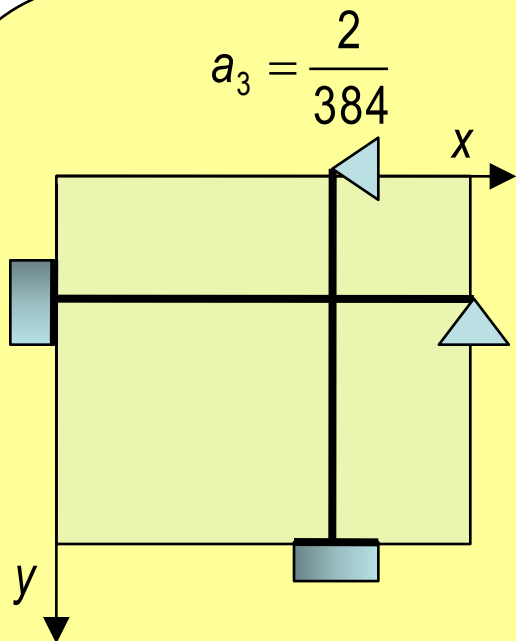
$$\frac{a_3}{a_3} \cdot \frac{q_x}{q_y} = \frac{q_x}{q_y} = \left(\frac{L_y}{L_x} \right)^4 = \lambda^4$$

$$q_x = \lambda^4 \cdot q_y \qquad q_y = \frac{1}{\lambda^4} \cdot q_x$$

$$q = q_x + q_y$$

4. case: Slab with two fix and two simple edges

4. eset: Szemben lévő peremlein befogott-csuklós lemez



$$w_x = \frac{2}{384} \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = \frac{2}{384} \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

$$q_x \cdot L_x^4 = q_y \cdot L_y^4$$

$$\frac{a_3}{a_3} \cdot \frac{q_x}{q_y} = \frac{q_x}{q_y} = \left(\frac{L_y}{L_x} \right)^4 = \lambda^4$$

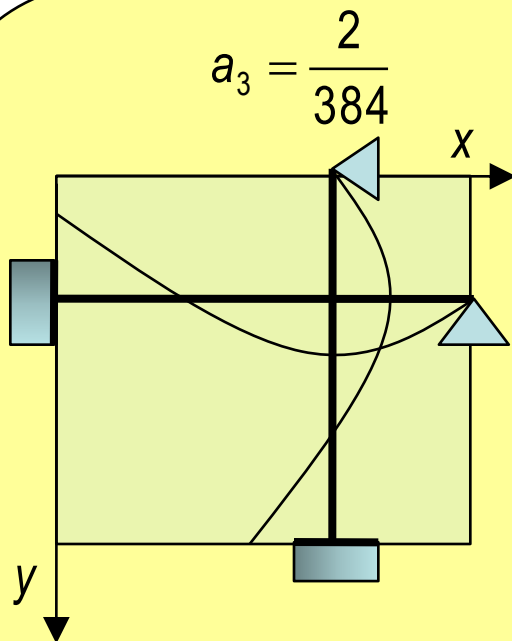
$$q_x = \lambda^4 \cdot q_y \qquad q_y = \frac{1}{\lambda^4} \cdot q_x$$

$$q = q_x + q_y$$

$$q_x = \frac{\lambda^4}{1 + \lambda^4} \cdot q \qquad q_y = \frac{1}{1 + \lambda^4} \cdot q$$

4. case: Slab with two fix and two simple edges

4. eset: Szemben lévő peremlein befogott-csuklós lemez



$$w_x = \frac{2}{384} \cdot \frac{q_x \cdot L_x^4}{E \cdot I} = w_y = \frac{2}{384} \cdot \frac{q_y \cdot L_y^4}{E \cdot I}$$

$$q_x \cdot L_x^4 = q_y \cdot L_y^4$$

$$\frac{a_3}{a_3} \cdot \frac{q_x}{q_y} = \frac{q_x}{q_y} = \left(\frac{L_y}{L_x} \right)^4 = \lambda^4$$

$$q_x = \lambda^4 \cdot q_y \qquad q_y = \frac{1}{\lambda^4} \cdot q_x$$

$$q = q_x + q_y$$

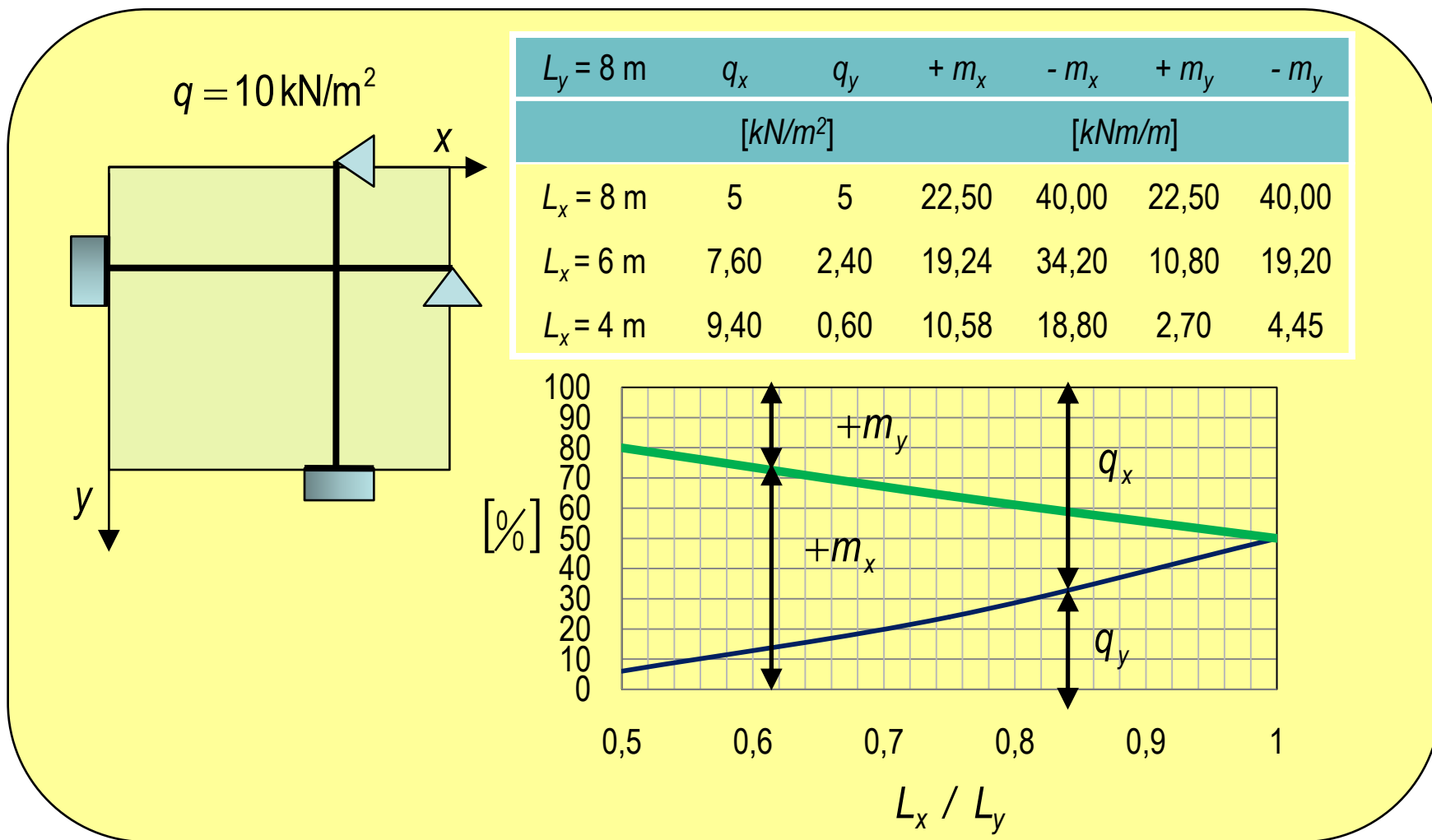
$$q_x = \frac{\lambda^4}{1 + \lambda^4} \cdot q \qquad q_y = \frac{1}{1 + \lambda^4} \cdot q$$

$$+m_{x,y} = \frac{9}{128} \cdot q_{x,y} \cdot L_{x,y}^2$$

$$-m_{x,y} = \frac{16}{128} \cdot q_{x,y} \cdot L_{x,y}^2$$

4. case: Slab with two fix and two simple edges

4. eset: Szemben lévő peremein befogott-csuklós lemez



Reinforced Concrete Structures II.

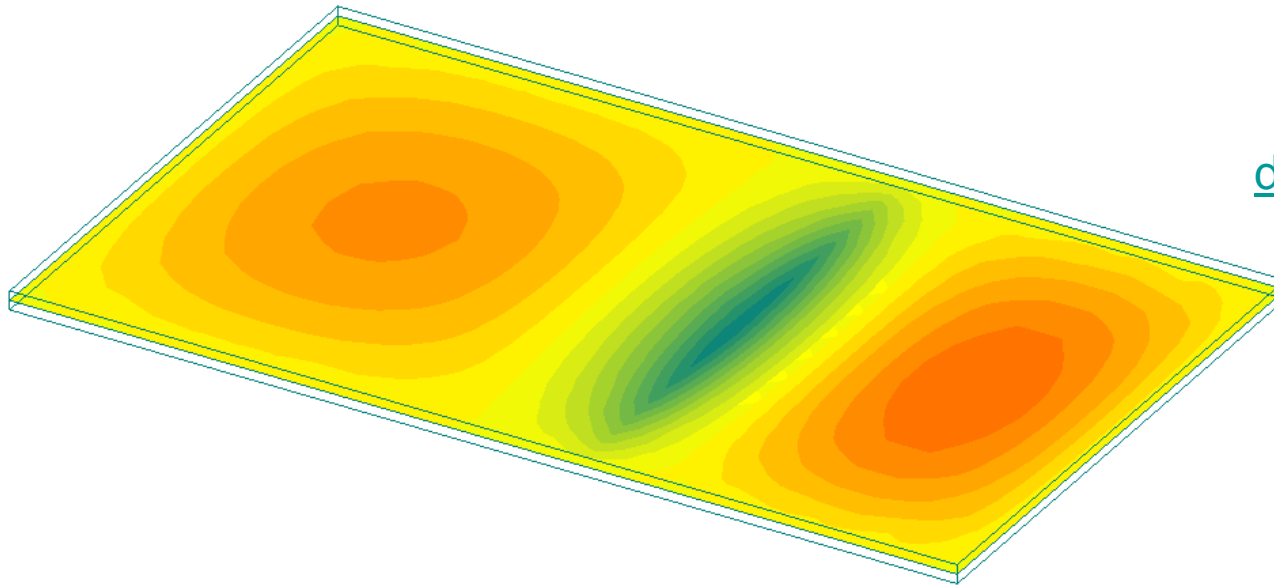
II.

Vasbetonszerkezetek II.

- Kétirányban teherviselő lemez nyomatókai - Tartósáv módszer -

Köszönöm a figyelmet!

Dr. Kovács Imre PhD
tanszékvezető
főiskolai tanár



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