Modeling of Plastic Matrix-Fiber Interaction in Fiber Reinforced Concrete

Imre Kovács
Technical University of Budapest

Supervisors:
Franz-Josef Ulm, LCPC, Paris
György L. Balázs, TU Budapest

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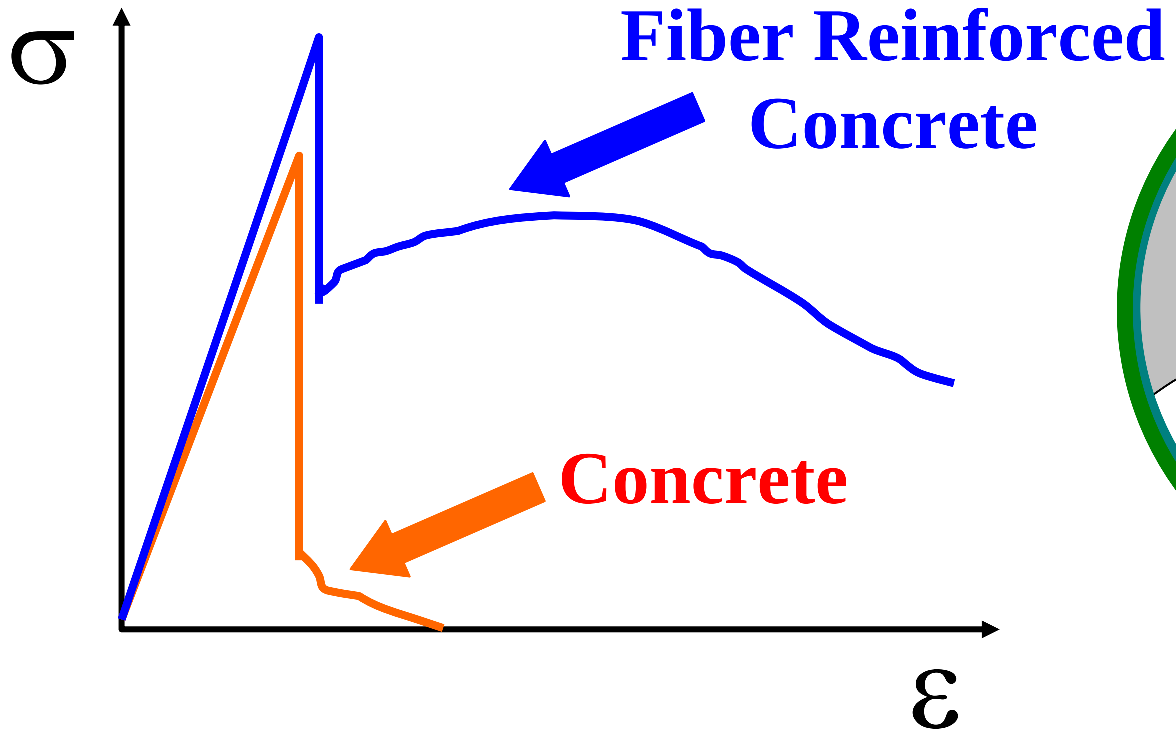
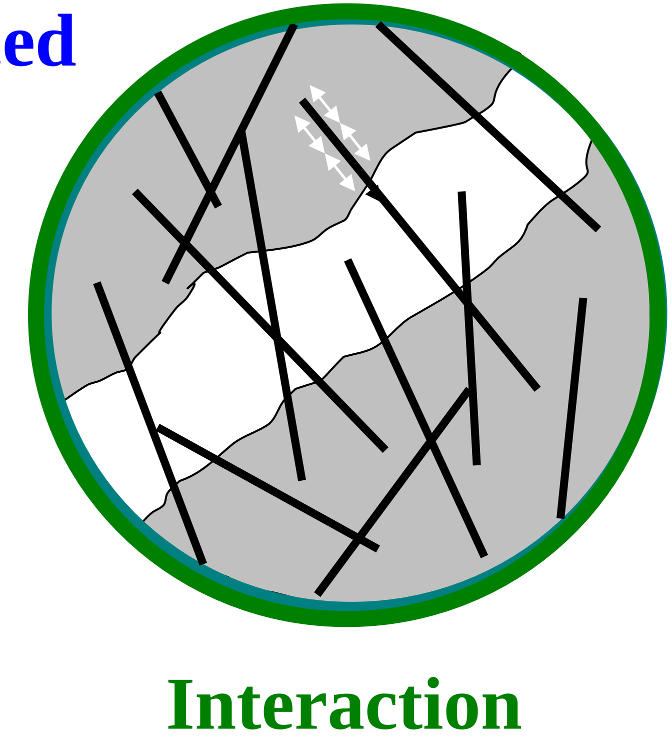
Plan

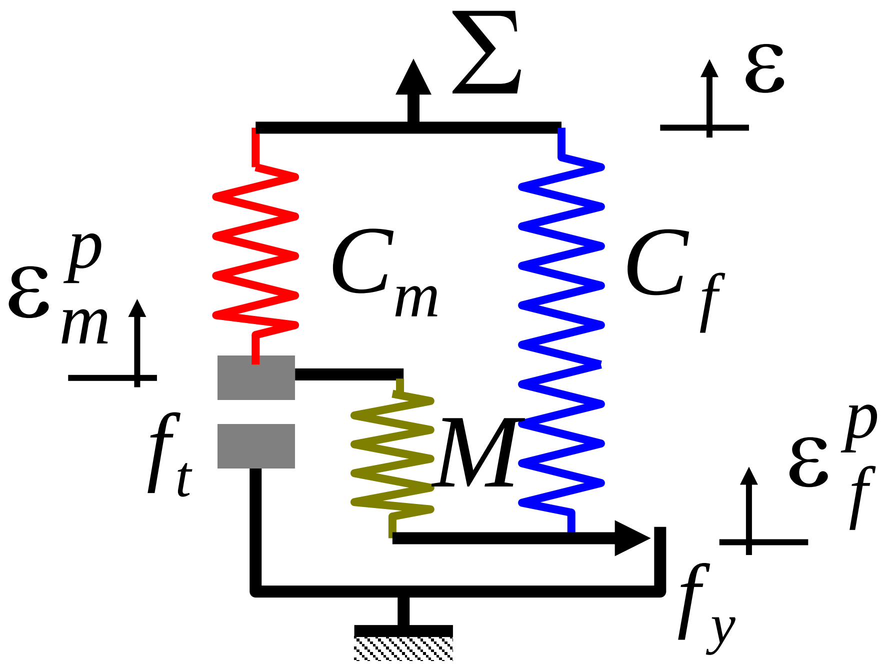
- **Problem**
- **1-D rheological model**
- **Application to RPC 200**
  (Reactive Powder Concrete)
- **Perspectives**
- **Conclusions**

Problem:





1-D Rheological model

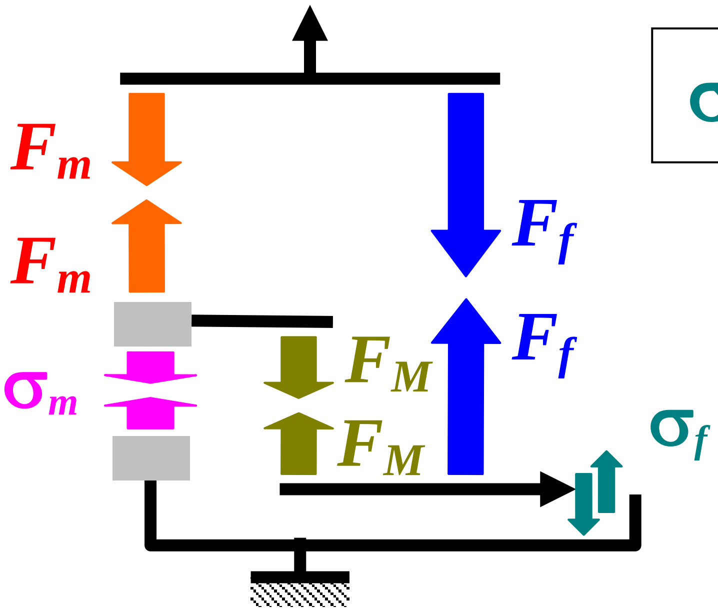


Force-flow

$$\Sigma = F_m + F_f$$

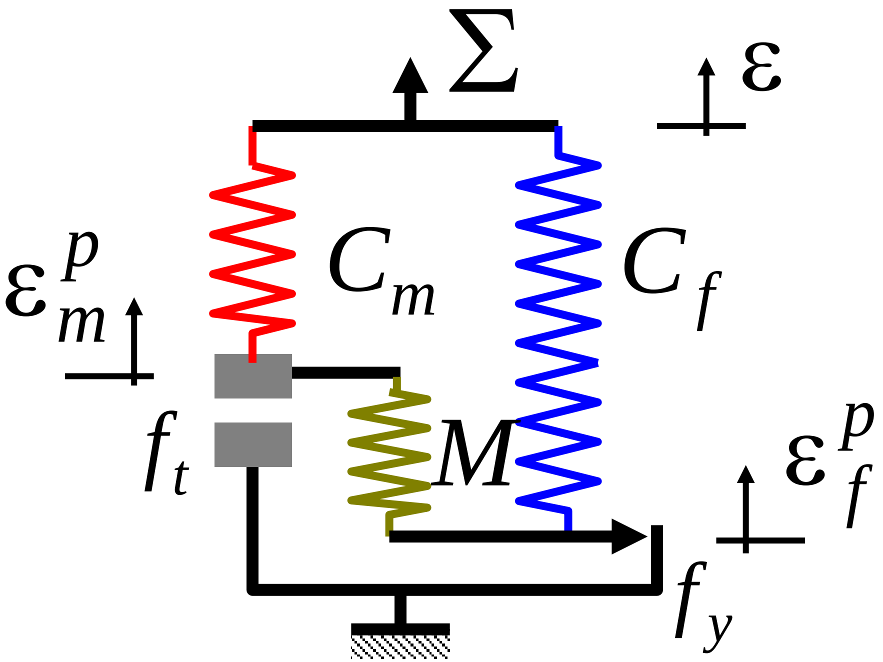
$$\sigma_m = F_m - F_M \le f_t$$

$$\sigma_f = F_f + F_M \le f_y$$



$$\Sigma = \sigma_m + \sigma_f$$

Stress-strain relation



$$\Sigma = \left(C_m + C_f\right)\varepsilon - C_m \varepsilon_m^p - C_f \varepsilon_f^p$$

$$\sigma_m = C_m\left(\varepsilon - \varepsilon_m^p\right) - M\left(\varepsilon_m^p - \varepsilon_f^p\right)$$

$$\sigma_f = C_f\left(\varepsilon - \varepsilon_f^p\right) + M\left(\varepsilon_m^p - \varepsilon_f^p\right)$$

Admissible stress states

Elastic state:

$$f = \max\{ f_m , f_f \} < 0$$

Matrix cracking

loading function:

$$f_m = \sigma_m - f_t \leq 0$$

if:

$$f_t / f_y > C_m / C_f$$

Fiber yielding

loading function:

$$f_f = \sigma_f - f_y \leq 0$$

if:

$$f_t / f_y < C_m / C_f$$

Matrix cracking

Plastic Evolution Law:

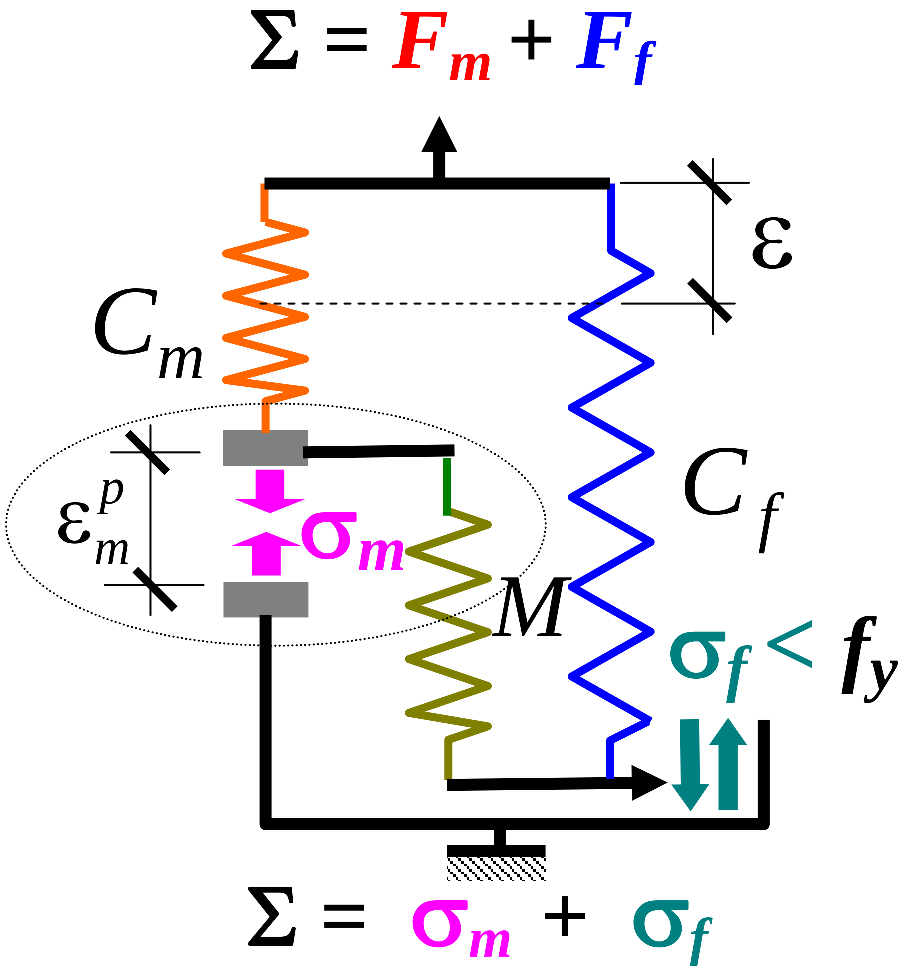
$$\sigma_m = C_m\left(\varepsilon - \varepsilon_m^p\right) - M\varepsilon_m^p = 0$$

$$\varepsilon_m^p = \frac{C_m}{C_m + M}\varepsilon$$

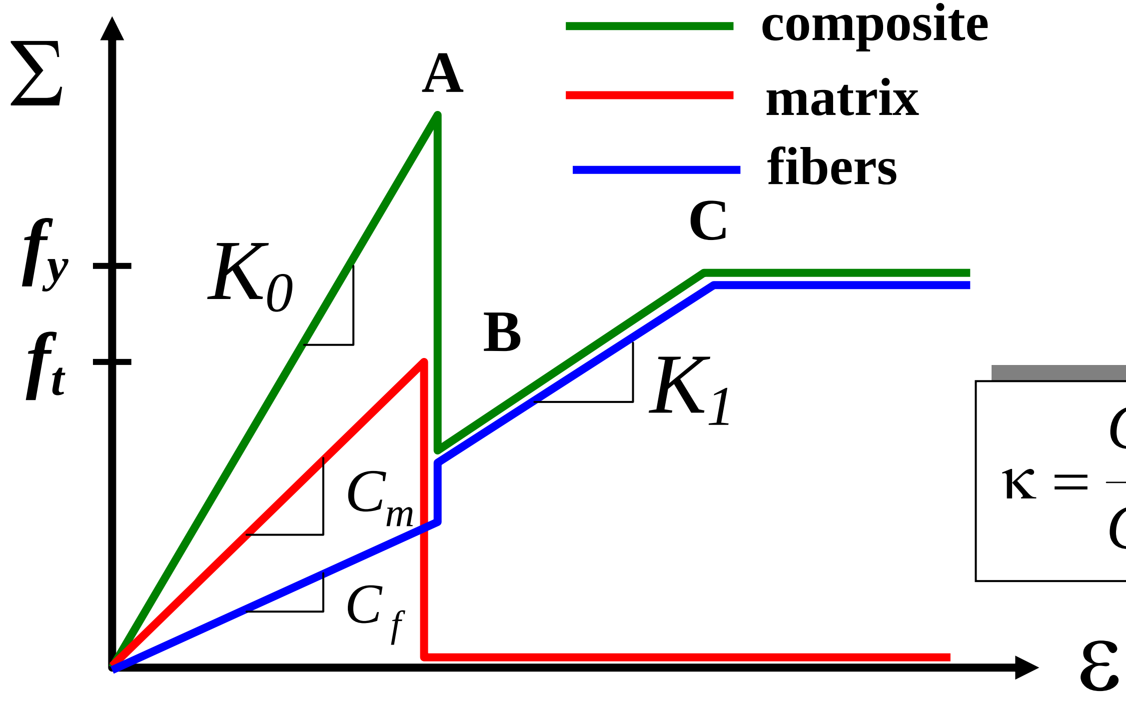
Tangential Stress-Strain Relation:

$$d\sigma_m = 0$$

$$d\sigma_f = d\Sigma = \left(C_f + M\frac{C_m}{C_m + M}\right)d\varepsilon$$



Stress-strain relation



$$\Sigma^A = f_t(1+\kappa)$$
$$\sigma_m^A = f_t$$
$$\sigma_f^A = f_t\kappa$$

$$\Sigma^C = f_y$$
$$\sigma_m^C = 0$$
$$\sigma_f^C = f_y$$

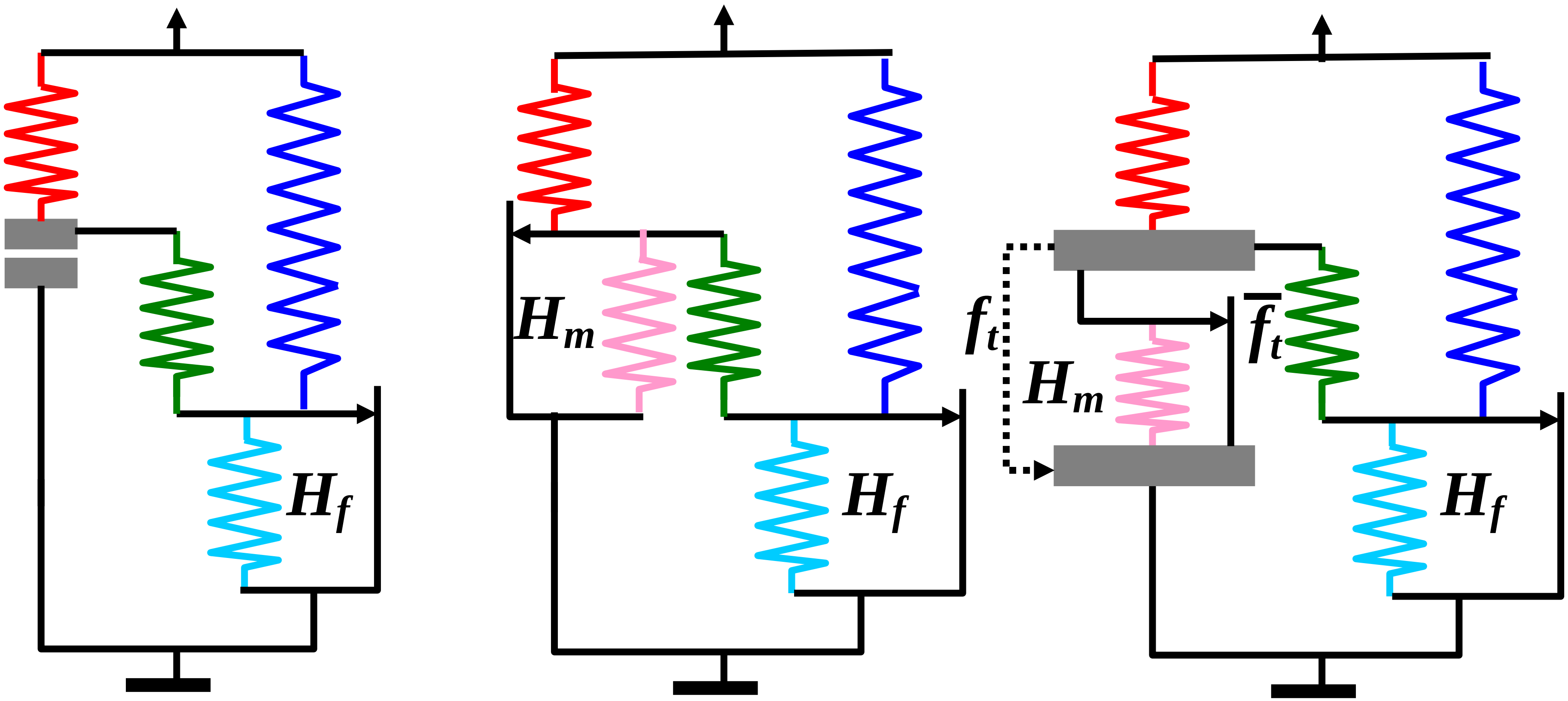
$$\kappa = \frac{C_f}{C_m}$$

$$\Sigma^B = \Sigma^A - f_t\frac{C_m}{C_m + M}$$
$$\sigma_m^B = \sigma_m^A - f_t = 0$$
$$\sigma_f^B = \sigma_f^A + f_t\frac{M}{C_m + M}$$

$$K_0 = C_m + C_f$$

$$K_1 = K_0 - C_m\frac{C_m}{C_m + M}$$

Model extension: softening/hardening

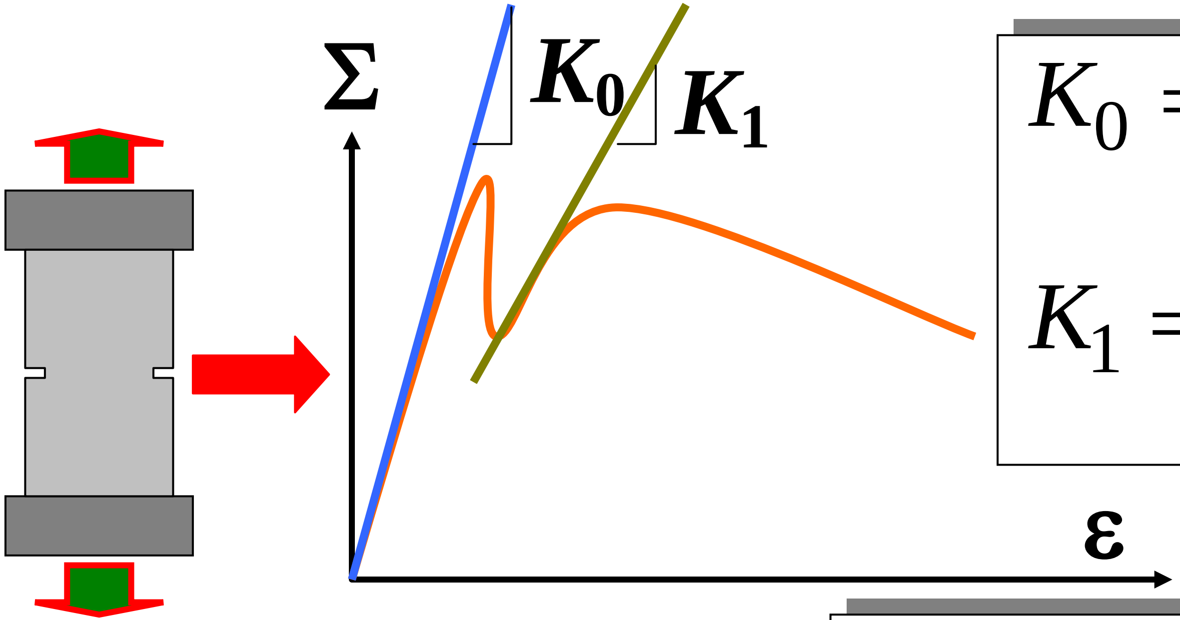


Elastic-brittle matrix
Elastic-plastic fibers

Elastic-plastic matrix
Elastic-plastic fibers

Elastic-brittle matrix with residual strength

Asymptotic determination of coupling modulus *M* in case of FRC



$$K_0 = C_m(1+\kappa)$$

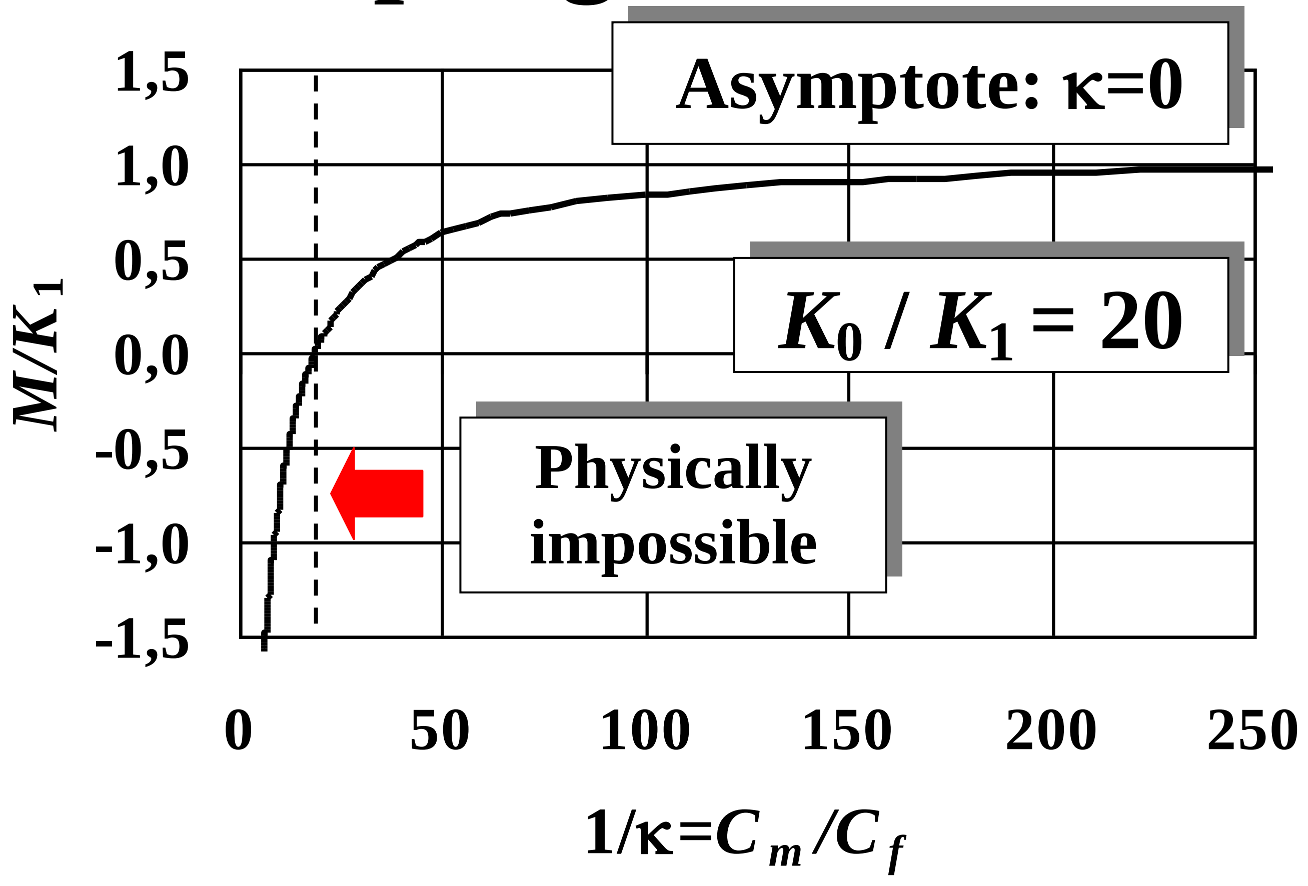
$$K_1 = C_m\left(\kappa + \frac{M}{C_m + M}\right)$$

DATA: K_0, K_1

GIVEN: $\kappa = C_f / C_m$

$$M = K_1 \frac{1+\kappa(1-K_0/K_1)}{(1+\kappa)^2(1-K_1/K_0)}$$

Asymptotic behavior of coupling modulus *M*



Asymptotic determination of coupling modulus *M* in case of FRC

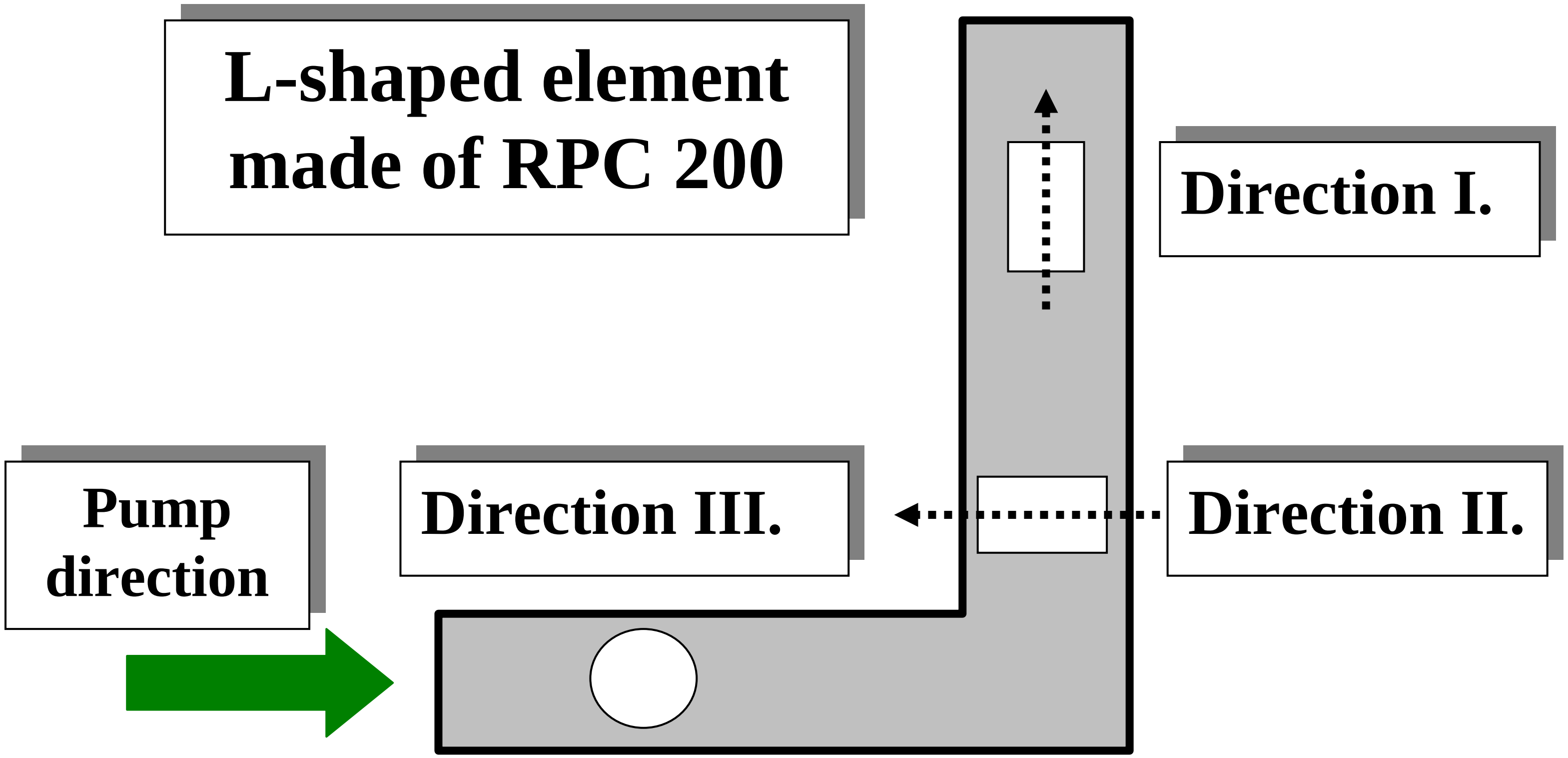
$$\lim_{\kappa \to 0} K_0 = \lim_{\kappa \to 0} C_m (1 + \kappa) = C_m$$

$$\lim_{\kappa \to 0} K_1 = \lim_{\kappa \to 0} C_m \left(\kappa + \frac{M}{C_m + M} \right) = \frac{C_m M}{C_m + M}$$

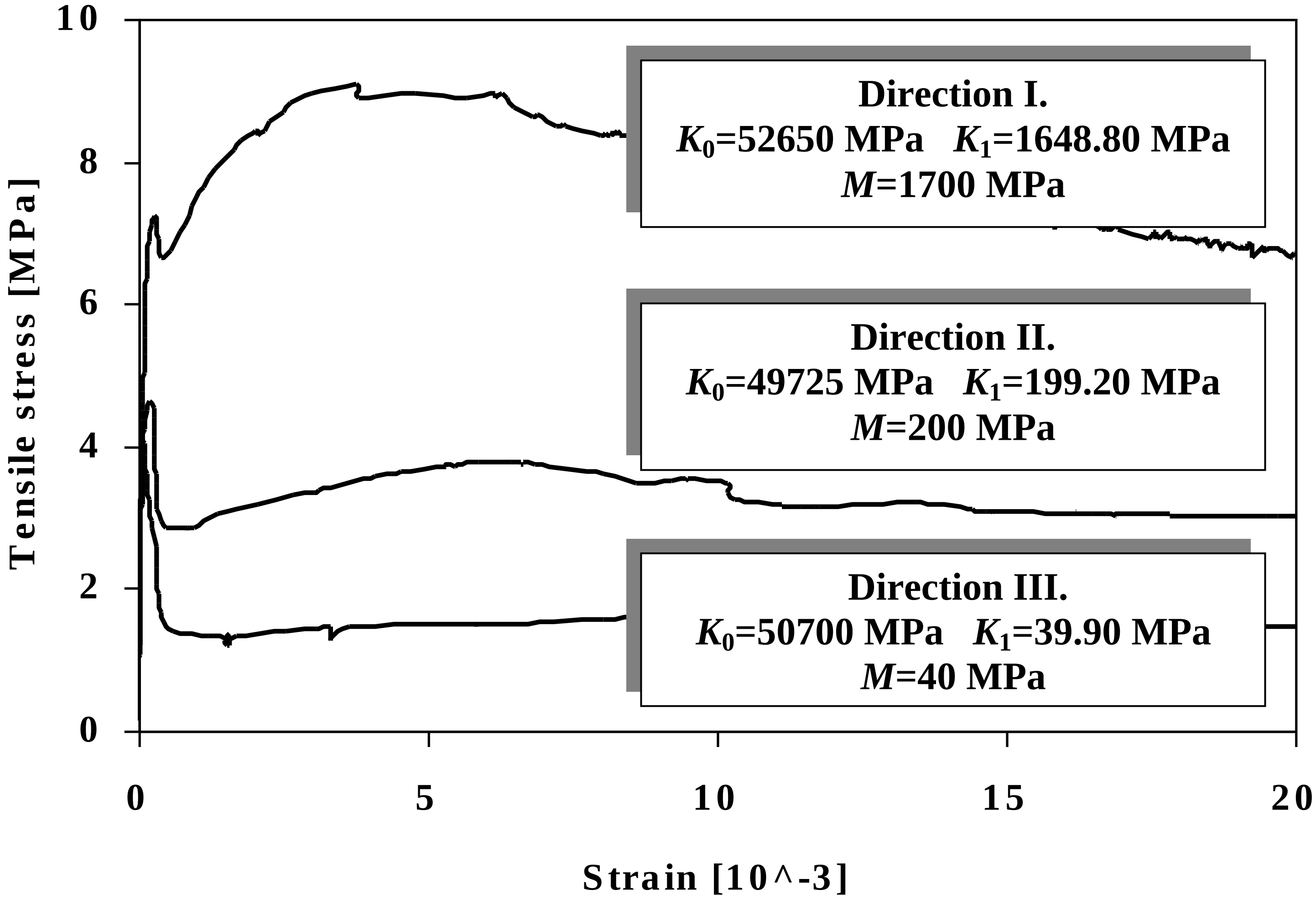
$$\lim_{\kappa \to 0} M = \frac{K_0 K_1}{K_0 - K_1}$$

Experimental study

Direct tensile test on notched specimens



Model application to RPC 200



Perspectives

- **Material characterization**

  $M = M$(fiber type, concrete, etc.)

- **Energetic approach**

  Motivation: ■ 3-D extension

- **Orientation of the fibers**

  Solution: ■ Fluid-mechanics

- **Numerical implementation**

  Problems: ■ Brittle fracture

  ■ Matrix-fiber interaction

Conclusions

- **1-D material model for FRC taking into account plastic matrix-fiber interaction is worked out**
- **Advantage:**
  - **Clear physical significance**
  - **Little number of material tests:**

    in the simplified 1-D case accessible by 1 standard material test
- **Because of the simplicity of the 1-D rheological model it can easily be extended with softening/hardening phenomena**